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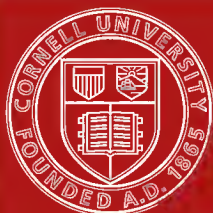
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A text-book on shades and shadows, and p



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A TEXT-BOOK
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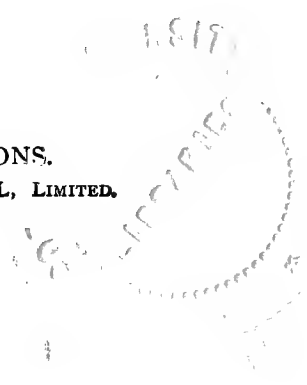
*PREPARED FOR THE USE OF STUDENTS IN
TECHNICAL SCHOOLS*

BY

JOHN E. HILL, M.S., M.C.E.,
Professor of Civil Engineering, Brown University.

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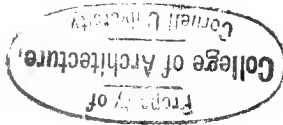


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PREFACE TO THE FIRST EDITION.

THIS text-book is designed to furnish a short course in Shades and Shadows, and One-Plane Perspective, for students who have studied the elements of Descriptive Geometry. Prominence has been given to the fact that the subjects treated are branches or applications of Descriptive Geometry and follow that subject in logical sequence.

The author is greatly indebted to many friends who have kindly assisted in the preparation of the work, and especially to Messrs. C. W. Comstock and C. W. Sherman, Instructors in Civil Engineering, Cornell University.

The method of perspective given is that elaborated in "Modern Perspective" by Professor W. R. Ware, and the method of shading used is to be found in the works of M. Jules Pillet. To many works, but especially to those mentioned, the author desires to express his obligation.

J. E. H.

ITHACA, N. Y., March 1, 1894.

PREFACE TO THE SECOND EDITION.

FOR this edition the book has been thoroughly revised and in the main rewritten. The most notable changes are in the notation of Shades and Shadows; the extension and addition of certain problems; and the addition of examples of shaded objects showing the application of the principles of shading.

My thanks are due Professor H. S. Jacoby, of Cornell University, for many suggestions.

J. E. H.

PROVIDENCE, R. I., July 1, 1896.

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SHADES AND SHADOWS.

CHAPTER I.

GENERAL PRINCIPLES, DEFINITIONS AND CONVENTIONS.

Shadows.

1. SHADES AND SHADOWS is primarily an application of Descriptive Geometry. (See §§ 16 and 19.)

2. The subject defined. The subject treats of one of the means employed by which the representation of any object may be given the appearance of reality. This result is here accomplished by giving certain tints or colors to one or both of the orthographic projections of the object upon the co-ordinate planes, and by the determination of its shadow either upon those planes, upon the object itself, or upon both.

Shading or tinting is treated in the Appendix.

3. Shades and Shadows in Nature. Non-luminous objects are seen by the light which they reflect and refract.* It is evident that without light an object is invisible. It is also evident that if an object be illuminated by but one source of light, *e. g.*, the sun, that the portion of the object towards the sun will appear light or bright, and the portion on the oppo-

* The detailed reasons for some of the statements in this chapter may be found in any of the larger text-books on Physics.

site side, or out of the light, will appear dark; also that various portions will appear to have different tints depending upon the relative positions which they bear to the sun. On a sphere the various tints blend and there are no sharp contrasts; on an hexagonal prism the tints on the several sides are sharply contrasted.

Because of this variation in the gradation of tints or colors in the appearance of objects, we are able to distinguish one from another, and were it not for this phenomenon all objects would appear flat, *i. e.*, like a plane.

In many cases we may determine by analogy the form of an object from its shadow; *e. g.*, one would never mistake the shadow of a tree for that of a building. If then the proper shading be given to the representation, and the shadow be determined, we have a picture of the object.

4. Sunlight The light from the sun is transmitted to the earth along slightly curved and slightly divergent lines, because the atmosphere is not homogeneous and the sun is not infinitely distant. We may consider, however, for drafting purposes that the lines of light are parallel right lines, as the error caused by such assumption is not appreciable for any drafting that would be made.

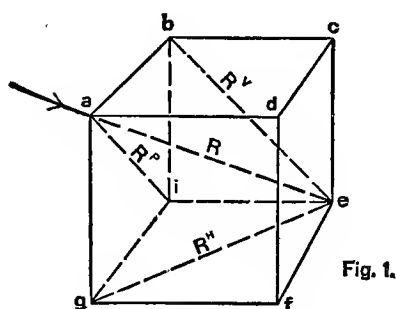
5. Conventions. The majority of draftsmen desiring uniform results have adopted the following conventional method of representation. It is unnecessary to discuss the scientific accuracy of these assumptions; let it be taken for granted that they supply the want and give the desired results.

Conventions.

(a) The source of light is considered to be a fixed point situated at an infinite distance from the object to be represented.

(b) All lines of light emanating from the source are parallel right lines.

(c) The source of light is further limited in position by being so placed with reference to the coördinate planes that a line of light forms one of the diagonals of a cube, two of whose faces are the coördinate planes. If the spectator were in the first dihedral angle looking at V , the source of light would be at an infinite distance above and behind his left shoulder; the direction of a line of light would be the diagonal of the cube above mentioned running from the near upper left vertex to the far lower right vertex. Fig. 1 gives a per-



spective view of a cube, two of whose faces, $gf ei$ and $bcei$, are in H and V respectively, and the diagonal R through a and e gives the direction of a line of light to which all lines of light are parallel.

(d) All lines of sight for either coördinate plane are parallel right lines perpendicular to that plane.

(e) All objects situated in the second, third, or fourth dihedral angles are considered invisible.

(f) All objects are to be considered opaque.

6. Rays. Lines of light are called *Rays of light*, or simply *Rays*. (See § 5, sec. (b).)

7. Plane of Rays. Any plane containing a ray of light is a *Plane of rays*, for such a plane contains an infinite number of lines parallel to the ray, and consequently an infinite number of rays.

8. Pencil of Rays. Any collection of rays other than a plane is called a *Pencil of rays*. (By some authors "cylinder of rays.")

9. Line of Shade. *The Line of Shade* of a surface or solid is the locus of the points of tangency of all rays which can be drawn tangent to that surface or solid. NOTE: The word tangent as used in this connection means *tangent* in its geometric sense, and also *touching without immediately intersecting*. The general idea of tangent rays is, however, always maintained, because the rays determining the line of shade do not intersect the object before touching, and do not immediately intersect after touching. (See Fig. 1.) The line of shade of the cube is composed of the six edges fd , dc , cb , bi , ig , and gf , determined by six planes of rays passed through them, and is an example of "touching without immediately intersecting." If a plane of rays were passed through the edge ag it would immediately intersect the cube, consequently ag is not a part of the line of shade. The line of shade of a sphere is a great circle determined by a *tangent pencil* (cylinder) of rays.

The line of shade divides the object into two parts, one illuminated, or in the light, and the other unilluminated, or in the dark; the light being excluded from the unilluminated part by the object.

The line of shade of a plane surface is the perimeter of the surface; of a plane figure, the lines of that figure.

10. Umbra. The *Umbra* or indefinite shadow of an object is that portion of space from which light is excluded by the

object. The umbra of a point is a line; of a right line is a plane, etc. The umbra of a surface or solid is bounded by the pencil of rays tangent to the object.

11. Shadow. The *Shadow* of an object is the intersection of the umbra with the surface of any object. If no object is situated in the umbra of an object there is no shadow. A shadow is always a point, line, or surface. (See Plate II, Fig. 11.) The shadow of the lower edge, $e b$, of the parallelopiped is partly on the three front faces and one side face of the under prism and partly on H . Care must be taken to obtain the complete shadow in every problem.

12. Primary and Secondary Shadows. A distinction is made between the *primary* shadow, or, briefly, the shadow, and the *secondary* shadow. The former is the intersection of the umbra with the first object which it meets, and the latter is a subsequent intersection. In many problems the secondary shadow is found in order to more easily determine the primary shadow.

13. Line of Shadow. The *Line of Shadow* is the perimeter of the shadow, or the intersection of the tangent pencil of rays with the object on which the shadow is cast.

14. Shade. The *Shade* is that portion of the *object* from which light is excluded by the object itself, and which would not be illuminated were any part of the object removed. The shade is bounded by the line of shade. (See § 9.)

15. It follows from §§ 9 and 13 that *the line of shadow is the shadow of the line of shade.*

16. Oblique projection. Since shadows are determined by lines (rays) which are oblique to the coördinate planes, the method is one of oblique projection, and the problems presented are similar to those of Descriptive Geometry involving

intersections. This fact should be constantly in mind when studying this subject. (See § 1.)

17. Conventional direction of rays. Since a ray forms the diagonal of a cube as shown on Fig. 1 (see § 5, sec. (c)), the *projections* of a ray make angles of 45° with $G. L.$ The ray itself makes angles of $35^\circ 15' 52''$ with H, V , or a profile plane; this angle is called θ for convenience.

Trigonometric functions of θ .

(See Fig. 1. Each edge of the cube is unity.)

$$\sin \theta = \frac{1}{\sqrt{3}} = 0.57735.$$

$$\cos \theta = \frac{\sqrt{2}}{\sqrt{3}} = 0.81649.$$

$$\tan \theta = \frac{1}{\sqrt{2}} = 0.70711 = \cos 45^\circ = \sin 45^\circ.$$

$$\sec \theta = \frac{\sqrt{3}}{\sqrt{2}} = 1.22475.$$

$$\cos 2 \theta = \frac{1}{3} = 0.33333.$$

18. Advantages of conventional direction of rays. By using the conventional direction of rays the construction of many problems may be shortened; *e. g.*, to find the shadow of the point a (Plate I, Fig. 3): As $a^H u$ is greater than $a^V u$, from a^H lay off $a^H p = a^V u$, draw $p a_{s1}$ parallel to $G. L.$, and $a^H a_{s1}$ making 45° with $G. L.$ The intersection of the two lines, a_{s1} , is the shadow of a required, because $a^V u$ is the distance of the point above H , and if a right-angled triangle be constructed with $a^H a = a^V u$ for the altitude and a ray for the hypotenuse, the base in H is $a^H a_{s1}$.

In certain problems it may be advantageous to disregard the conventional position of the source of light and assume some other position in order to bring out more clearly, either

the character of the object or a particular detail. Because such cases are rare and their treatment requires mature judgment no departure is made in this work from the ordinary method of procedure. Usually a change in the position of the object rather than in the position of the light will give the desired result.

19. The general problem. In general the shadow of an object is found by passing a pencil of rays tangent to the object and determining the intersection of this pencil with the object which receives the shadow; this method involves, first, the determination of the line of shade, and second, its shadow. When the line of shade is a right line and the surface receiving the shadow is a plane, the problem is simple and may be stated thus: To find the intersection of a plane passed through a given line with a given plane. When the line of shade is a curve and the surface receiving the shadow is curved the problem is more difficult; stated in Descriptive Geometry language it is: To find the intersection of any surface with a cylinder.

In problems relating to curves or surfaces of the second degree, the axes or diameters of parts of the shadows may often be found and the solution greatly simplified. As a last resort a number of rays may always be passed through the line of shade and their intersections with the object receiving the shadow found. This method should not be employed however, unless necessary, as the great number of construction lines required increases the chances for error and has a tendency to mar the appearance of the drawing.

The line of shade in many cases is readily determined by inspection. When this cannot be done, or uncertainty exists in regard to lines probably composing the line of shade, the shadow of the entire contour line of the object may be found

and the parts which cast shadows within the perimeter of the shadow rejected. The general test is, that no part of the line of shade casts a shadow within the line of shadow.

20. Notation. The following notation will be employed throughout Shades and Shadows:

(a) The *projections* of a point on the coördinate planes will be designated by small letters with the exponents ^h and ^v representing the horizontal and vertical projections respectively; e. g., the projections of the point *a* are a^h and a^v .

(b) The *projections* of the *revolved* position of a point will be designated by the letter of the point with the subscript ₁, ₂ or ₃ and the exponents ^h and ^v; e. g., a_1^h and a_1^v .

(c) The *projection* of a point upon a *new coördinate plane* will be designated by the letter of the point with the exponent ^h₁ or ^v₁, ^h₂ or ^v₂; e. g., a^{h_1} or a^{v_1} .

(d) The *shadow* of a point on one of the coördinate planes will be designated by the letter of the point with the subscript _s; e. g., a_s . When the *shadows** of a point on both coördinate planes are required they will be designated by the letter of the point with the subscripts _{s1} and _{s2}; e. g., a_{s1} and a_{s2} .

(e) The *projections of the shadow* of a point on a surface other than the coördinate planes will be designated by the letter of the point with the exponents ^h and ^v and the subscripts _{s1} and _{s2}, etc., e. g., a_{s1}^h , a_{s1}^v . When only one projection of the shadow is required it will be designated by the letter of the point with the subscript _s; e. g., a_s .

(f) A *line* will be designated by a capital letter, and its projections, revolved position and shadows by the exponents or subscripts used for points. See (a), (b), (c), (d), (e).

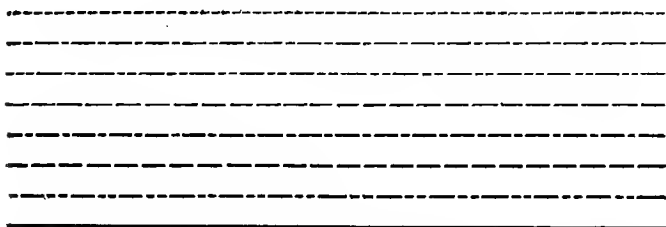
(g) The *trace* of a plane will be designated by a capital letter preceded by a capital *H* or *V*; e. g., *HP* or *VP*.

* Primary and Secondary shadows (see § 12) usually.

(h) The *trace* of a plane in a *revolved* position will be designated by the letter of the plane with the subscript ₁, ₂ or ₃, and preceded by a capital *H* or *V*; e. g., HP_1 or VP_1 .

(i) The *trace* of a plane on a *new* coördinate plane will be designated by the letter of the plane, preceded by a capital *H* or *V* with the subscript ₁, ₂ or ₃; e. g., H_1P or V_1P .

21. Drafting Conventions.



(1) Given and required lines, if visible, are represented by heavy full lines; if invisible, by heavy long dashes.

(2) Auxiliary lines, visible and invisible, are represented by light dashes. Projecting lines and lines indicating the paths of rotated points are short dashes and other auxiliary lines long dashes.

(3) Traces of given and required planes, if visible, are represented by heavy full lines; if invisible, by one long dash and two short dashes, also heavy.

(4) Traces of auxiliary planes—planes used as a means for solving the problem—are represented by light broken lines consisting of one long dash and two short dashes alternately.

(5) Intersections of given and required planes, if visible, are represented by heavy full lines; if invisible, by one long dash and one short dash, also heavy.

(6) Intersections of auxiliary planes, or of auxiliary with given or required planes, are represented by light broken lines consisting of one long dash and one short dash alternately.

(7) The long dashes should measure about one-eighth of an inch and the short dashes about one twenty-fourth of an inch, *i. e.*, about one-third as long as the long dashes. The spaces between the dashes should be very short.

(8) New ground lines are represented by heavy full lines.

(9) All objects of which the shadows are to be found are considered opaque.

Recapitulation. Given and required lines, traces, and intersections, if visible, are distinguished from the invisible in form only, the weight remaining the same. When visible all of them are represented by full lines.

When given or required, but invisible, they differ from the auxiliary in weight only, the form remaining the same. Traces have one long dash and two short dashes, intersections one long dash and one short dash, and other lines are composed of either long or short dashes.

All given and required lines, etc., are heavy, and all auxiliary lines are light.

22. Problems in Drafting Room. The work in the drafting room may often be greatly facilitated and improved by assigning problems in which the positions of points, etc., are designated by means of coördinates, as in analytic geometry.

The position of a point in space is known when its distance from three planes (no two of which are parallel) is known, hence to designate the position of a point in a problem in shades and shadows (descriptive geometry), we may refer it to H , V , and a profile plane, P . Following the notation of analytic geometry, $G. L.$ is the axis of x , the horizontal trace of P is the axis of y , and the vertical trace of P is the axis of z . The point a with the following reference, $x = 2''$, $y = 3''$, $z = 6''$, means that the point is in the first dihedral angle $2''$ to the right of P , $3''$ in front of V and $6''$ above H . The point b

with $x = 3''$, $y = -3''$, $z = -4''$, means that the point is in the third dihedral angle $3''$ to the right of P , $3''$ behind V , and $4''$ below H . The designation of lines and planes readily follows. It is preferable to assume that the reference profile plane (P) passes through the left border line of the sheet upon which the drawing is to be made.

CHAPTER II.

Shadows of Right Lines.

23. Problem I. *To find the shadow of a line on the coördinate planes.* Plate I, Fig. 3.

Analysis. The shadow of a line is determined by the intersection of a plane of rays through the line with the object receiving the shadow; or, in other words, the shadow is the locus of the intersections of all rays passed through the line with the object receiving the shadow. The object in this case is the coördinate planes, hence if the shadows of any two points of the line on H and on V be found and joined by a line, the required shadow is determined. Or the shadow of one point of the line may be found on H and on V , and joined with the H and V traces of the line respectively, for since the traces of a line are points in the coördinate planes, each is its own shadow on the plane in which it is situated, and the shadow of the line is determined as before.

Construction. Let M be the given line. Take any two points of the line as a and b and through each pass a ray. The projections of the rays pass through the projections of the points and make angles of 45° with $G. L.$ The intersection (trace) of the ray through a with H is a_{s1} and with V is a_{s2} . The intersection (trace) of the ray through b with H is b_{s2} and with V is b_{s1} . Joining a_{s1} b_{s2} we have the shadow of M on H , and joining a_{s2} b_{s1} , the shadow of M on V . The parts of the shadow on H behind V and on V below H are secondary (see

§ 12), hence the shadow of M on the coördinate planes is the line $a_{s1}n_s b_{s1}$ or M_{s1} .

In this problem, as in many others, by finding the secondary shadow the solution is simplified. The method of § 18 might have been employed with advantage.

Another Solution. Since a_{s1} and b_{s2} are the H traces of rays passed through two points of the line M and a_{s2} and b_{s1} are the V traces of the same rays, $a_{s1} b_{s2}$ and $a_{s2} b_{s1}$ are the H and V traces respectively of a plane of rays passed through M , and hence are the shadow of M on H and V . The visible portion of these traces and hence the primary shadow is the broken line M_{s1} .

24. Problem II. *To find the shadow of a plane surface on the coördinate planes.* Let the given surface be a triangle. Plate I, Fig. 4.

Analysis. The line of shade of the surface is the perimeter of the surface (§ 9) and the line of shadow is the shadow of the perimeter (§ 15), hence the shadow of a triangle is determined by the shadows of its three sides. If we find the shadows of the vertices of the triangle we will have the shadows of two points in each side, hence the shadows of the vertices determine the shadow of the triangle.

Construction. Let abc be the given triangle. Find the shadows of the vertices. The shadow of a is a_s on H , of b is b_{s1} on V , and of c is c_s on H , all found as in Prob. 1, § 23. Join $a_s c_s$ for the shadow of ac . As a_s and c_s are on H while b_{s1} is on V , it is necessary to find the shadow (secondary) of b on H , or of a and c on V , in order to determine the direction of the shadow of ab and of cb on H . The shadow of b on H is b_{s2} , hence $a_s m_s$ and $c_s n_s$ are the primary or visible shadows of ab and cb respectively on H . To complete the shadow of the triangle draw $m_s b_{s1}$ and $n_s b_{s1}$, the primary shadows of ab

and cb respectively on V . The required line of shadow is $r_s m_s b_{s1} n_s c_s a_s$.

25. Problem III. *To find the shadow of a solid on the coördinate planes.* Let the solid be a cube. (Simplest case.) Plate I, Fig. 5.

Analysis. The line of shadow of a solid is the shadow of the line of shade (§ 15), hence first determine, if possible, the line of shade. In complicated problems in solids it is sometimes difficult if not impossible to determine the line of shade without virtually finding the shadows of various lines of the object. The criterion to be employed is: A line of the solid is a part of the line of shade when its shadow is a part of the perimeter of the shadow of the solid. A line of the solid is *not* a part of the line of shade when its shadow falls *within* the perimeter of the shadow of the solid. For rectilinear solids we may usually determine the position of the line of shade by inspection if we remember the definition of § 9.

The line of shade of the cube is found by inspection to be the edges dj, jg, gf, fb , and the edges in H, ab and ad which coincide with their shadows. The proof of this statement will be given below. Having determined the line of shade, proceed as in Probs. I and II.

Construction. Given the cube as shown with one face in H and the other faces perpendicular or parallel to V . The shadow of the edge dj is $d^H j_s$, of jg is $j_s g_s$, of gf is $g_s f_s$, of fb is $f_s b^H$, found as in Prob. I by joining the shadows of the two points of each edge. The shadows of the edges ab and ad coincide with the lines. The line of shadow is, therefore, $d^H j_s g_s f_s b^H a^H d^H$ on H . The visible shadow is bounded by the line $d^H j_s g_s f_s b^H c^H d^H$.

Proof of statement in analysis:—Suppose the edge ej to be part of the line of shade, its shadow is $e^H j_s$ which falls

within the line of shadow, hence ej is not a part of the line of shade. (See § 19.)

A plane of rays through ej immediately intersects the cube, hence ej is not a part of the line of shade. (See § 9.) In the same manner it may be shown that the edges ae , ef , etc., are not parts of the line of shade.

NOTE: From the solution of this problem we see several facts which are principles of geometry. (a) If a line is parallel to a plane its shadow on that plane is parallel to the line. (b) Parallel lines cast parallel shadows on the same plane, etc.

26. Problem IV. *To find the shadow of a vertical prism on the coördinate planes.* Plate I, Fig. 6.

Analysis. The analysis of this problem is similar to that of Prob. III.

Construction. The line of shade is found by inspection to be the edges cj and af , three sides of the upper base jk , kl , and lf , and the two sides of the lower base ab and bc ; which statement may be proved as in Prob. III. The shadow of cj is $c^H j_{s1}$, of jk is $j_{s1} n_s k_{s1}$ (see Prob. I), of kl is $k_{s1} l_{s1}$, of lf is $l_{s1} f_{s1}$, of af is $f_{s1} p_s a^H$, of ab is $a^H b^H$, and of bc is $b^H c^H$. The complete line of shadow is now determined. The visible portion of the shadow is bounded by the line $c^H j_{s1} n_s k_s^1 m_s d^V p_s a^H e^H d^H c^H$.

27. The shadow of the upper base of the prism on H and on V is shown and may be used as the solution of the problem: To find the shadow of a horizontal polygon on one or both of the coördinate planes. The shadow (visible) of the polygon on the coördinate planes is the line $j_{s1} n_s k_{s1} l_{s1} f_{s1} q_s g_{s1} j_{s1}$. (See Prob. II.)

28. Problem V. *To find the shadow of a plane surface on an oblique plane.* Let the surface be a triangle. Plate I, Fig. 7.

Analysis. See analysis of Prob. II, § 24. The shadow of

a point on a plane is the intersection of a ray through the point with the plane, and of a line on a plane is the intersection of a plane of rays through the line with the plane. Hence two methods of solution are available: either find the shadows of the vertices of the triangle by means of rays, or the shadows of the sides of the triangle by means of planes of rays. In general, the first method is the simpler.

Construction. Let abc be the given triangle, and Q the plane on which the shadow is cast. To find the shadow of c on Q pass a vertical *plane of rays* Z through the point. This plane intersects the plane Q in the line $M(M^v, HZ)$. Hence c_s , the intersection of a ray through c with M , is the shadow of c required. In a like manner pass the vertical planes of rays Y and X through a and b respectively. As the vertical planes of rays Z , Y , and X are parallel, their intersections with the plane Q are parallel, hence U^v and T^v may be drawn through the intersection of the V traces parallel to M^v . A ray through a intersects U at a_s , and a ray through b intersects T at b_s . The shadow of the triangle is, therefore, $a_s b_s c_s$.

29. The student should constantly have in mind the fact that the solutions of the following problems are exactly alike: "To find the shadow of a point on a plane" and "To find the intersection with a given plane of a line passed through a given point and parallel to a given line." "To find the shadow of a right line on a plane" and "To find the intersection with a given plane of a plane passed through a given line and parallel to a given line."

30. Problem VI. *To find the shadow of one line upon another line.* Plate I, Fig. 8.

Analysis. If one line casts a shadow upon another line a plane of rays through the first line intersects the second line, otherwise the two lines are in parallel planes of rays and the

second cannot receive a shadow from the first. If a plane of rays be passed through each line these planes intersect and the line of intersection is a ray, because the line of intersection of any two planes of rays is a ray. Hence, if the shadows of the two lines on the same object be found and a ray passed through their intersection, the required shadow and the point which casts the shadow is determined by the intersections of the ray with the two lines. A limiting case of the above is when the two lines are in the same plane of rays, in which case the second line is the shadow of the first line.

Construction. Let M and N be the given lines. The shadows of these lines on H are M_s and N_s respectively, which intersect at the point o_{s2} . A ray through o_{s2} is the line of intersection of the planes of rays which determined the shadows of the lines. This ray intersects N at o_{s1} and M at o ; hence the shadow of M on N is o_{s1} .

31. Problem VII. *To find the shadow of a hexagonal abacus on a vertical hexagonal prism.* Plate I, Fig. 9.

Analysis. See analysis of Prob. V, § 28. Only the shadow of the abacus on the prism is considered. It is evident from an inspection of the figure that the lower edges ef , fa , and ab of the abacus compose the line of shade for the purpose named. Having found the line of shade the problem becomes: To find the shadow of a line on a plane (§ 28).

Construction. Let M, N, O , etc., be a vertical prism, and $ab c$, etc., be the abacus. As the faces of the prism are vertical the intersection of the H projection of a ray through any point of the line of shade, as f , with the H projection (base) of the prism gives the H projection of the shadow of that point on the prism. Through f pass a ray: it intersects the prism at f_s ; hence f_s is the shadow of f on the prism. Through k, j, g, a , and i pass rays: they intersect the prism at k_s, j_s, g_s, a_s , and

i_s , respectively; hence $k_s f_s j_s g_s a_s i_s$ is the line of shadow required of which the portion $g_s a_s i_s$ is invisible on V projection and all is invisible on H projection. The points k, j, g , and i were determined as follows: Take the line of shade fa : it is evident that its shadow falls on three faces of the prism, PQ , QL , and LM ; hence one point of fa must cast a shadow on Q and one point on L . Since Q is perpendicular to H through Q^n draw the H projection of a ray: it intersects $f^n a^n$ at j^n ; hence j is that point of fa which casts a shadow on Q . Similarly for the points g, k , and i . In general, the shadow of a line changes direction whenever the object receiving the shadow changes direction. As the line of shade of a rectilinear solid is a broken line, its shadow on an object usually changes direction whenever the line changes direction.

32. Problem VIII. *To find the shadow of a vertical prism surmounted by a parallelopiped.* Plate II, Fig. 11.

Analysis. This problem is a combination of Probs. IV and VII. The parallelopiped overhangs the prism which contains a recess.

The line of shade of the parallelopiped may be determined by inspection, but that of the prism can only be approximated in this manner, for a shadow is cast by the parallelopiped upon that which would otherwise be a part of the line of shade. Hence the shadow on the prism is determined before the line of shade of the prism. As the *visible* shadow is the only part which is usually required for a representation of an object, in this problem the invisible shadow is not determined, except as much as is necessary for the solution.

Construction. The line of shade of the parallelopiped is composed of the edges ac, cx, xd, de, eb , and ba . The shadow of ac is $a_s p_s c_s$, of cx is $c_s l_s$ (the remainder of the shadow is invisible), of xd is invisible, of ba is $b_s a_s$. A ray through e

pierces the prism at e_s , which is the shadow of e required. Since the edge ed is perpendicular to V and the left front face of the prism is parallel to V , the shadow of ed on this face must pass through e_s at an angle of 45° with $G.L.$; hence $e_s f_s$ is the shadow. Another method for finding f_s is that given in Prob. VII, § 31. The shadow of fd is invisible. The edge eb is parallel to $G.L.$, and three of the front faces of the prism are parallel to V ; hence the shadow of eb on these faces is parallel to $G.L.$ Through e_s draw $e_s g_{s1}$ and $j_s k_{s1}$ parallel to $G.L.$, since the two front faces are in the same plane. The point i of the edge eb casts the shadow i_s on the edge D of the prism (see Prob. VII, § 31); hence $i_s g_{s2}$ parallel to $G.L.$, and limited by a ray through the edge B , is the shadow on the inner face of the recess and $i_s j_s$ is the shadow on the right face of the recess. The remainder of the edge eb , or $k b$, casts a shadow $k_{s2} b_s$ on H . This completes the line of shadow of the parallelopiped. The line of shade of the prism is composed of parts of the edges A , B and C and parts of the lower base. As the shadows of m and n , two points of the lower base, on H are within the H projection of the parallelopiped, it is evident from an inspection of the figure that the entire shadow of this base is within the projection; hence there is no visible shadow of this part of the line of shade, and only the three edges need be considered. The shadow of A is $m_s k_{s2}$, limited by a ray through k , and the visible portion is $u_s k_{s2}$; of B is $g_{s2} w_s$ on the inner face of the recess, and limited by a ray through g and the base of the prism; of C is $n_s q_s o_s$, limited by a ray through o and the visible portion is $r_s q_s t_s$. The shadow on the prism is seen only in V projection, and is bounded by the line $f_s e_s g_{s1}$, part of B^v , part of $n^v m^v$, $w_s g_{s2} i_s j_s k_{s1}$, part of A^v , part of $b^v d^v$, thence down C^v to f_s . On the coördinate planes the visible shadow is bounded by the line $b_s a_s p_s c_s l_s m^v t_s q_s r_s c^H u_s k_{s2} b_s$.

33. Problem IX. *To find the shadow of a chimney and a dormer on a roof.* Plate II, Fig. 12.

Analysis. The solution of this problem shows the use of a profile plane as the roof is assumed parallel to $G. L.$

If a plane of rays be passed through a point its intersection with any plane is a locus of the shadow of the point on that plane, also a ray through the point is a locus of the shadow of the point; hence the intersection of these loci is the shadow of the point on the plane. Similar reasoning applies to the shadow of a line on a plane.

Construction. Let the plane W be the plane of the roof parallel to $G. L.$, E, F , etc., be the chimney perpendicular to H , and $b f, e t$, etc., be the dormer, which is supposed to be faced with stone. The relative positions are shown clearly by the three projections.

The line of shade of the chimney is $E a d c F$. Through a pass a plane of rays parallel to $G. L.$, it intersects the roof W in G , found as follows: Project the point a on a profile plane P , revolve into V and obtain a_1^P , through a_1^P draw $a_1^P x_1^V$, making an angle of 45° with $G. L.$: this is the revolved position of the intersection of the plane of rays through a with the plane P , since a plane of rays parallel to $G. L.$ bisects the first dihedral angle. Revolve back to original position and thus determine x , which is one point in the line of intersection G , which may now be drawn through x parallel to $G. L.$ A ray through a intersects G at a_s , which is the shadow of a on the plane W . Since E is perpendicular to H , a plane of rays through E is perpendicular to H ; hence the H projection of its shadow or $a^H a_s^H$ is inclined at an angle of 45° with $G. L.$ The H projection of the visible shadow is $a^H f^H$ and $f_s^H a_s^H$, as the part $f^H f_s^H$ is on the shadow of the dormer. The V projection of the shadow of E runs from m along the roof to the face of the dormer, thence

up the face of the dormer along E_s^v , thence becomes invisible but again appears in the line $f_s^v a_s^v$. A similar construction applies to the remainder of the line of shadow. The line of shade of the dormer is $o i b f$ and the rear part (see figure) of et , the shadow of which is found by the method used before.

Another method of solution is that given in Prob. V, § 28, in which a profile plane is not used.

CHAPTER III.

Shadows of Curves.

34. Problem X. *To find the shadow of a sphere on the coördinate planes.* Plate II, Fig. 13.

Analysis. The pencil of rays tangent to a sphere is evidently a cylinder having for its axis a ray through the center of the sphere and for its right section a *great circle* of the sphere. The plane of the line of shade is therefore a meridian plane of the sphere perpendicular to a ray. Having determined the line of shade the problem then becomes: to find the shadow of a circle on the coördinate planes.

Construction. Given the sphere A as shown. Project the sphere on a new V plane coinciding with a vertical plane of rays through the center. The new V projection is A^{v_1} . Through the center o pass a ray: it intersects H at o_{s1} ; hence $o^{v_1}o_{s1}$ is the projection of the ray on V_1 . (This projection might also have been obtained by passing a line through o^{v_1} making an angle of $35^\circ 15' 52''$ with G_1 . L_1 .) The line $c^{v_1}o^{v_1}d^{v_1}$ perpendicular to $o^{v_1}o_{s1}$ is the intersection of the plane of the line of shade with V_1 .

Passing rays through c and d , and using the new V plane, the shadow $c_s d_s$ of a diameter of the line of shade is determined, and hence a diameter (axis in this case) of the shadow. As the diameter dc is in a plane perpendicular to H , the diameter ab parallel to H is perpendicular to it; hence the shadow of ab , or $a_s b_s$, is the diameter (axis in this case) which

is conjugate to $c_s d_s$ of the shadow. The points a_s and b_s are the intersections of $b_s o_{s1} a_s$ drawn perpendicular to $c_s d_s$ with the H projections of rays through a and b . We know that $c_s d_s$ and $b_s a_s$ are not only diameters, but axes of the shadow, because $a b$ is parallel to H and $c d$ is in a plane perpendicular to $a b$ and to H ; hence the shadows of these lines are perpendicular to each other (see § 35). Constructing an ellipse with $c_s d_s$ and $b_s a_s$ as axes the shadow of the sphere on H is determined.

As rays are equally inclined to the coördinate planes and the plane of the line of shade is perpendicular to the rays and the line of shade is a circle, the shadow of the sphere on V must be an ellipse equal in all respects to that on H . Hence, since the shadow of the center on V is o_{s2} , lay off on the V projection of the ray through the center, $o_{s2} j_s = o_{s1} c_s = o_{s1} d_s = o_{s2} f_s$, and on a line through o_{s2} perpendicular to $j_s o_{s2} f_s$ lay off $o_{s2} g_s = o_{s2} e_s = o_{s1} b_s = o_{s1} a_s$. An ellipse constructed on the axes $j_s f_s$ and $g_s e_s$ is the shadow of the sphere on V . The shadow of the sphere is bounded by the line of shadow $c_s b_s p_s g_s f_s m_s c_s$, which is the solution sought.

35. Proposition I. *The oblique projections (or shadows) of any pair of rectangular diameters of a circle or conjugate diameters of an ellipse are conjugate diameters of the oblique projection (or shadow) of that circle or ellipse.*

36. Proposition II. *If two surfaces intersect each other in two curves one of which is a curve of the second degree, both curves are of the second degree, and the surfaces are of the second degree.*

This proposition for use in the subsequent problems may be stated thus:* When we cut off by a plane and remove a portion of a surface of the second degree, such as a cylinder,

* See "A Course in Shades and Shadows," by William Watson.

cone, etc., the shadow of the section cast upon the interior surface so exposed is a plane curve, and consequently one of the second degree.

37. *To construct an ellipse having given any pair of its conjugate diameters.* Plate II, Fig. 14.

Construction. Let ab and cd be the conjugate diameters of an ellipse intersecting at o , the center. With o as a center and radius equal to the semi-longer diameter describe a circle. Draw nom perpendicular to ab at o , and join mc and nd . Draw any line as wrq parallel to nom and intersecting ab and the circle, then draw trp parallel to doc and qp and wt parallel to mc and nd . The points p and t are points of the ellipse.

38. *To construct the axes of an ellipse having given any pair of its conjugate diameters.* Plate I, Fig. 10.

Analysis and Construction. Let $a_s b_s$ and $c_s d_s$ be conjugate diameters of an ellipse in H . Draw $G. L.$ through b_s parallel to $c_s d_s$; then $G. L.$ is tangent to the ellipse.

As only *one* ellipse can be constructed on a given pair of conjugate diameters, as $a_s b_s$ and $c_s d_s$, and as these lines may be considered the shadows of a pair of rectangular diameters in an infinite number of ellipses, we may assume for the purpose of our demonstration that these lines are the shadows of a pair of rectangular diameters of a circle situated in a plane which passes through $G. L.$ On this assumption the diameter of the circle which casts the shadow is equal to $c_s d_s$. NOTE: It is not necessary to determine in any case the actual ellipse or circle which casts the shadow, as we wish to find only the magnitude and direction of the axes of the shadow; also, the rays of light are not assumed in the conventional direction for the same reason.

If the radii of the circle be produced they will intersect

$G. L.$ and the shadows of the produced radii will run from the intersections on $G. L.$ to o_s , the shadow of the center. A certain pair of rectangular radii cast rectangular shadows, which are the axes of the elliptical shadow. As each radius and its shadow intersects $G. L.$ at the same point, the required pair of radii and their shadows may each be inscribed in a semi-circle of the same radius and having a common diameter in $G. L.$

To find the semi-circles revolve the plane of the circle casting the shadow about $G. L.$ into H , the circle takes the position as shown with center at o_1 , join $o_1 o_s$, bisect $o_1 o_s$ and draw the perpendicular $g i$ intersecting $G. L.$ at i , with i as a center and radius equal to $i o_s = i o_1$ describe a circle which gives the semi-circles required. Draw $o_1 f$ and $o_1 e$, $f o_s p_s$ and $e o_s m_s$, also $q_1 q_s$ and $n_1 n_s$ parallel to $o_1 o_s$, then $q_s p_s$ and $n_s m_s$ are the required axes.

39. *To find the position (not magnitude) of the axes of an ellipse, having given any pair of its conjugate diameters.** Plate II, Fig. 15.

Construction. Let $o a$ and $o b$ be the semi-conjugate diameters of an ellipse. Produce $o a$ to c making $o a \times a c = o b \times o b$.

Draw $m a n$ parallel to $o b$ and through o and c describe a circle having its center in $m a n$. Draw $o m$ and $o n$, which are the indefinite axes of the ellipse.

40. **Problem XI.** *To find the shadow of the edge of a hemispherical shell on the interior surface.* Plate III, Fig. 16.

Analysis. The line of shade or part of the edge which casts a shadow on the interior surface is determined by the diameter of the line of shade of the sphere which is parallel to H (in this case) and is the semi-circumference of the edge.

The shadow is a conic section (§ 36), in this case the arc of

* See Salmon's Conic Sections.

a circle, because the only plane curve which can be cut from a sphere is a circle.

Construction. Given the hemispherical shell $a b c$, with the plane of the edge parallel to H . The diameter $b c$ perpendicular to the rays and parallel to H is a diameter of the line of shade and also of the shadow, as it joins two points b and c on the line of shadow. (See Prob. X, § 34.) The radius $o a$ is perpendicular to $b c$, hence its shadow $o a_s$, on the plane of the shadow of the edge, is a semi-diameter of the shadow conjugate to $b c$; but $b c$ and $o a_s$ are perpendicular, hence the *axes* of the projection of the shadow on H are determined. The V projections of the axes $b c$ and $o a_s$ are conjugate diameters of the V projection of the shadow. To find the shadow of a on the interior, either pass a meridian plane of rays through a and revolve parallel to V about an axis perpendicular to H through the center of the shell, or project on a V plane of rays through G_1, L_1 . The V_1 projection of a is a^{v_1} . Through a^{v_1} draw R^{v_1} , making an angle of $35^\circ 15' 52''$ with G_1, L_1 : the intersection $a_s^{v_1}$ with the contour is the V_1 projection of the shadow required; hence a_s is the shadow of a on the interior.

Short Construction. Draw $a_s^{v_1} o^{v_1}$ and project $a_s^{v_1}$ on $a^{v_1} m$ at n . The angle $m o^{v_1} a_s^{v_1} = 2 \text{ angle } o^{v_1} a^{v_1} a_s^{v_1}$. Therefore $o^{v_1} n = o^{v_1} a_s^{v_1} \cos 2 \theta = \frac{r}{3}$. Hence we may at once lay off $o^n a_s^n = \frac{r}{3}$ and construct an ellipse. The shadow of the edge on the interior is $b a_s c$ found by constructing ellipses on axes in H projection and conjugate diameters in V projection. Only the shadow in H projection is visible.

41. Problem XII. *To find the shadow of an oblique cylindrical surface on the coördinate planes and the shadow of the upper base on the interior of the surface.* Plate III, Fig. 17.

Analysis. The line of shade is determined by two tangent

planes of rays and two pencils (cylinders) of rays through the bases. The cylinder, being hollow, part of the shadow of the upper base is on the interior surface and part on the coördinate planes. To determine the elements of shade, pass a plane through the axis (or element) of the cylinder and a ray; this plane is parallel to the tangent planes, which are now readily determined.

The problem "to pass a *plane of rays* tangent to a cylinder or cone" is the same problem as "to pass a plane tangent to a cylinder or cone and parallel to a given line." From § 36 we know that the shadow on the interior is a conic section, in this case an ellipse, and hence a *plane* curve.

Construction. Given the oblique cylindrical surface with one base in and the other parallel to H , as shown.

Shadow on the coördinate planes. The shadow of a the center of the upper base on H is a_{s2} ; hence $b^H a_{s2}$ is the H trace of a plane of rays through the axis, because it contains the H trace of the axis and the H trace of a ray through a . The H traces of the two planes parallel to this plane and tangent to the cylinder are $c^H r_s$ and $e^H x_s$, and the elements of shade are therefore cd and ef , which cast shadows $c^H r_s d_s$ and $e^H x_s f_s$ respectively. The line of shade (edge) of the upper base is divided in two parts by the elements of shade, the shadow of fgd being on the coördinate planes and the shadow of fjd on the interior.

Draw the rectangular diameters df and jg : their shadows on V are conjugate diameters of the shadow of the upper base; hence, constructing an arc of an ellipse on $d_s f_s$ and $j_s g_s$ limited by the points f_s and d_s , we have the complete shadow of the surface on the coördinate planes.

Shadow on the interior surface. Since df is a diameter of the line of shade and contains two points of the shadow it is a

diameter of the shadow. The shadow of j , the extremity of the rectangular diameter gj , is j_s , found as follows: Through the element tj pass a plane of rays; it intersects the cylinder in the element wg ; a ray through j intersects wg at j_s , hence j_s is the shadow of j on the interior. A plane through fd and j_s is the plane of the line of shadow (see § 36), because the curve of shadow is a plane curve, and this plane contains three points of that curve.

Produce $j_s a$ to k on the element tj , then $j_s a = ak$ and df and $j_s k$ are conjugate diameters of the elliptical shadow. A semi-ellipse constructed on $d^H f^H$ and $a^H j_s^H$ is the H projection of the shadow, and a semi-ellipse on $f^V d^V$ and $a^V j_s^V$ is the V projection of the shadow. The visible shadow on the interior is bounded by the line $f^H j_s^H u d^H j_s^H f^H$, and on the coördinate planes by the line $c^H r_s y i^V x_s z c^H$.

42. *Another solution* for finding shadow on interior of the surface of preceding problem. Same figure.

Analysis. The intersection of the umbra of any curve with another curve is the shadow of the first curve on the second (see § 30).

Hence the intersection of a pencil of rays through the upper base of the cylinder with any other section is the shadow of the base on that section.

Construction. Pass any auxiliary plane through the cylinder as the horizontal plane X ; it cuts out the section B . The shadow of the upper base on the plane X is A_s , the circumference of a circle of radius equal to that of the base, since the plane of the base and X are parallel. A_s and B intersect at p_s and q_s , hence these points are two points of the required shadow. A sufficient number of planes to accurately determine the line of shadow must be employed.

43. **Problem XIII.** *To find the shadow of a conical surface*

on the coördinate planes and the shadow of the base on the interior of the surface. Plate III, Fig. 18.

Analysis. This problem presents a fact concerning tangent planes to cones which is often overlooked. The elements of shade are determined by two tangent planes of rays. As these tangent planes are planes of rays each must contain the ray the which passes through the vertex of the cone, hence this ray is line of intersection of the two planes. Using the conventional direction of rays (§ 5) this line of intersection or ray cannot be parallel to the coördinate planes, but must intersect them. The H and V traces of the tangent planes must pass through the corresponding traces of this line, hence the H traces of these planes cannot be parallel, and the V traces cannot be parallel. As in the preceding problems, the shadow on the interior is a conic section, in this case an ellipse.

Notice the fact that in this problem the line joining a and b is a *chord* and not *diameter* of the line of shadow as the line fd in the preceding problem (§ 41).

Construction. Let the conical surface be given as shown with axis perpendicular to H , and base the circumference of a circle parallel to H . To find the elements of shade pass a ray through the vertex o : it intersects the plane of the base at j ; hence ja and jb drawn tangent to the base are the intersections of the tangent planes of rays with the plane of the base, and oa and ob are the elements of shade. The shadows of oa and ob are $o_s t_s a_s$ and $o_s k_s b_s$ respectively. The shadows of any two rectangular diameters of the base as fd and ce give conjugate diameters of the shadow of the base. Hence an ellipse constructed on $f_s d_s$ and $c_s e_s$ as conjugate diameters is the shadow of the base on V . But as the line of shade A is divided in two parts by the elements of shade and the shadow of

the arc alb is on the interior, the shadow of the base on V is limited by the shadows of a and b . Hence $o_s k_s b_s d_s c_s a_s t_s o_s$ is the shadow on the coördinate planes required.

Shadow on the interior surface. The chord ab of the line of shade is a chord of the interior line of shadow, since it joins two points a and b of that shadow. To find diameters of the line of shadow revolve the meridian plane containing a ray about the axis of the conical surface parallel to V . The point l , one extremity of the diameter of the line of shade perpendicular to ab , takes the position l_1 , and a ray through l in revolved position is $l_1 l_{s_1}$, hence the revolved position of the shadow of l is l_{s_1} and the true position is l_s . The plane of the line of shadow, therefore, passes through the points a , b , l_s (see § 36). In revolved position (perpendicular to V in this case) this plane contains the line $a_1 b_1$ (vertically projected at g_1^v) and the point l_{s_1} , and intersects the revolved position of the element ol at n_1 , hence $l_{s_1} n_1$ is the revolved position of a diameter (axis) of the elliptical shadow. Bisect $l_{s_1} n_1$ at m_1 ; a chord of the cone $i_1 r_1$ through m_1 perpendicular to $l_{s_1} n_1$ and in the plane $a_1 b_1, g_1$ is the revolved position of another diameter (axis) of the shadow. Revolve back to the original position and we have the lines $l_s^v n^v, i^v r^v$ for conjugate diameters of the V projection, and $l_s^H n^H, i^H r^H$ for the axes of H projection of the shadow. Constructing arcs of ellipses thus determined and limited by the points a and b , we have the shadow on the interior, only the H projection of which is visible.

The method of § 42 might also have been employed for finding the interior shadow.

The small figure above Fig. 18 shows the shadow of a right circular cone on the coördinate planes.

44. Problem XIV. *To find the shadow of a horizontal circle on a diagonal plane.** Plate III, Fig. 19.

Analysis and Construction. A diagonal plane is a vertical plane inclined 45° to V . Let mn be a diameter, perpendicular to a ray, of the horizontal circle $abcd$: its shadow on the diagonal plane P is parallel to H and equal in length to mn , since mn is parallel to P . The shadow of the rectangular diameter pq on P is perpendicular to H and in length $= pq \tan \theta$. The V projection of the shadow of mn is $m_s^v n_s^v$ parallel to H and in length $= mn \sin 45^\circ = mn \tan \theta$, and the V projection of the shadow of pq is perpendicular to H and of length $= p_s q_s = pq \tan \theta$. But $mn = pq$, being diameters of the same circle; hence $mn \tan \theta = pq \tan \theta$, and the V projection of the shadow of the circle $abcd$ on the diagonal plane is a circle of radius $= r \tan \theta = \frac{r}{\sqrt{2}}$. This method may be used in Probs. XII, XIII, and XVI.

45. Problem XV. *To find the shadow of a surface of revolution on the coördinate planes.* First case: *The surface concave towards the axis.* Plate IV, Fig. 20.

Analysis. The typical surface of revolution taken is the torus or annulus.

Meridian plane method for determination of the line of shade. If a ray be projected (orthographically) on any meridian plane of the surface the projecting plane is a plane of rays (§ 7), and if planes be passed parallel to this plane and tangent to the surface the points of tangency are points of the line of shade (§ 9). The line of shadow on the coördinate planes is not a conic section, hence it is most readily determined by the

* This method for solving certain problems is due M. Jules Pillet, École Nationale des Ponts et Chaussées.

intersections of individual rays through the line of shade. As this part of the problem presents no difficulty, the line of shadow is not shown for the outer curve.

Construction. Given the torus as shown with axis perpendicular to H . Let Y be any meridian plane of the torus and po a ray intersecting this plane at o . The projection of po on the plane Y is wo , found by dropping a perpendicular from p to the plane. Revolve the plane Y about the axis of the torus, X , until parallel to V . The line wo takes the position w_1o ; hence the lines Q_1, M_1, P_1, N_1 parallel to this line and tangent in V projection to the contour of the torus are the revolved positions of the intersections of tangent planes of rays with the meridian plane Y , hence the revolved positions of four points of the line of shade are determined. Revolving back we have the points q, r, a and t of the line of shade. The points in the meridian plane Z found by the same method are l, k, m and n . In this manner all points of the line of shade, which is composed of the two curves $lqntl$ on the concave and $krmak$ on the convex side of the torus were determined.

Analysis of inner line of shade. It does not follow that a line of shade which fulfils only certain geometrical conditions casts a shadow; this is particularly true of surfaces convex towards the axis, like the inner part of a torus or the figure of Prob. XVI (Fig. 21). The complete shadow of $krmak$ on H is $k_s y i_{s2} m_s b_{s2} z k_s$. This line crosses itself at i_{s2} and b_{s2} which shows that the parts of the line of shade $irdi_{s1}$ and $bfa b_{s1}$ do not cast shadows on H . From i to r and from b to f the shadow is on the surface of the torus, while from d to i_{s1} and from a to b_{s1} the line is in shadow, and hence cannot cast a shadow.

From r to d and from f to a the H projection is convexed towards the axis, and if a plane of rays be passed parallel to Z

between these points and the inner circle of the torus it will cut the line of shade in four points.

Such a plane cuts a curve from the torus similar to that shown in Fig. 20a (the right and left ends of the curve are not shown). To this curve four rays may be drawn tangent as shown: one at a which is a point on the line of shade and casts a shadow on the curve near b , and the other three at b, c and d which are not points on a true line of shade, although geometrically so determined, because the rays intersect the torus before touching the curve. The true line of shade on the convex surface of the torus is the line $fbkir$. The shadow on the torus may be found by the methods of previous problems or by the secant plane method as indicated.

46. Problem XVI. *To find the shadow of a surface of revolution on the coördinate planes. Second case: The surface convex towards the axis. Plate IV, Fig. 21.*

Analysis. The typical surface taken is a scotia or base of a column, generated (in this case) by the revolution of two tangent arcs of circles joined to two lines about an axis situated in the plane of all the lines. (See figure).

Method of inscribed tangent surfaces. If two surfaces of revolution are tangent along the circumference of a circle their lines of shade intersect, for the pencils of rays tangent to the surfaces intersect at some point (or points) common to the two surfaces. As the line of shade of a sphere or cone is readily determined, inscribed tangent spheres or cones may be used to find the line of shade of the given surface. The problem is naturally divided into four parts:—First: To find the complete line of shade. Second: To find the shadow of the upper cylinder on the surface and on the coördinate planes. Third: To find the shadow of the double curved portion. Fourth: To find the shadow of the lower cylinder.

Construction. First: The line of shade of the upper cylinder is composed of the two elements yz and Y and half of each of the bases B and C . The line of shade of the lower cylinder is composed of the two elements W and Z , half of the lower base and parts of the upper base A limited by the elements W and Z and the shadow of the concave portion. *Line of shade of the double curved portion.* Let acd be any horizontal section of the surface cut by the horizontal plane T , also the line (circle) of tangency of an inscribed tangent sphere F . The radius of this sphere, ao , is determined by drawing iao in the principal meridian plane of the surface (M) perpendicular to a tangent (not shown) to the surface at a . The plane of the line of shade of the sphere is perpendicular to a ray (§ 34); hence its traces are perpendicular to the respective projections of a ray, hence $o^v b^v$ perpendicular to R^v is the V projection of the intersection of the plane of the line of shade of the sphere with the principal meridian plane of the surface M . The line ob pierces the plane of acd (circle of tangency) at b ; then $d^H b^H c^H$ perpendicular to R^H is the H projection of the intersection of the plane of the line of shade of the sphere with the plane of the circle of tangency, since the plane of the circle of tangency is parallel to H ; hence d and c , the points of intersection of dbc and this circle, are points of the line of shade required, for they are points of the line of shade of the sphere and of the line of tangency of the sphere and the given surface. In a like manner the points 1, 2, 3, 4, etc., were determined. The line of shade of the double-curved portion is $c2165d43c$. *Alternate method* by inscribed tangent cones. Let the circle acd (see before) be the line (circle) of tangency and also the base of an inscribed tangent cone. The vertex of the cone is e^v, o^H , found by producing the elements seen in V projection until they intersect the axis. (NOTE: If the base of the cone

were assumed above a horizontal plane through k the vertex would be below, instead of above, the base.) The shadow of the cone on the plane T of the base is found by drawing $e_s c$ and $e_s d$ tangent to the base (see § 43), hence ec and ed are the elements of shade. These elements intersect the line (circle) of tangency of the cone and given surface, or in other words intersects the base of the cone, at c and d ; hence these points are points in the required line of shade and coincide with the points found by the sphere method.

Second: *Shadow of the upper cylinder.* The shadows of the elements yz and Y are $y_s z_s$ and Y_s respectively. The shadow of the upper base B on V is part of an ellipse B_{s2} limited by the point z_s and $G. L.$, and constructed on the shadows $n_s q_s, r_s f_s$ of the rectangular diameters nq and rf . On H the shadow is B_{s1} , an arc of a circle, since B is horizontal. The shadow of the lower base C on V is C_{s2} , a part of an ellipse constructed on the shadows $w_s x_s, t_s u_s$ of the rectangular diameters wx and tu ; on H the shadow is composed of two arcs of the same circle whose center is p_{s2} and radius $= p_{s2} t_s = p t$, since C is horizontal. The shadow on the double-curved surface is determined by the method of § 42. Let Z be any horizontal secant plane; it cuts from the surface the circumference G . The shadow of C on this plane is found by constructing an arc with p_{s1} , the shadow of p on the plane, as a center and radius $=$ radius of C . This arc intersects G at m_s and l_s ; hence these points are two points of the required shadow. Other points were determined by the same method. The shadow of C on the surface is C_{s3} . The line of shade of the double-curved portion intersects C_{s3} at the points 2 and 5, at which points rays are tangent to the curve C_{s3} . Only that part of C_{s3} which is on the left of or above the line 2, 5 is on the outside of the scotia; the remainder of the line (on the

right of or below 2, 5) can only exist as a shadow on the supposition that the scotia is hollow and that the upper cylinder is removed; under these conditions this part of the shadow of C is on the interior.

Third: Through the points 1, 3, 4, and 6 of the line of shade of the double-curved portion draw tangent rays; these rays divide the line into four parts, as follows: The part above 1, 6 is on the interior or convex side of the surface, which is readily seen by drawing a ray tangent to the surface at some point above these points, *e.g.* in the meridian plane of rays through p , p_{s2} (see § 45). Such a ray is within the scotia and intersects the curve cut from the surface by the plane in a point on the side opposite the point of tangency. At the point 1 (and 6) the point of tangency and the point of intersection of the ray coincide, hence at this point the line of shade changes from the convex to the concave side of the surface. The parts 1 2 3 and 6 5 4 are on the concave side while the remainder (below 3, 4) is on the interior or convex side. We have shown that the shadow of C on the surface intersects the line of shade at the points 2 and 5; hence the part of the line of shade 2 1 6 5 (*i. e.* all above 2, 5) is in shadow and cannot cast a shadow even though the parts 2 1 and 5 6 are on the concave side of the surface, hence the only portions of the entire line which cast a shadow are the parts 2 3 and 5 4. The shadows of 2 3 and 5 4 are partly on the coördinate planes and partly on the surface. The shadow on the surface is determined by the method of §§ 30 and 42, thus: Find the shadows on H by individual rays; they are 2_s 3_s and 5_s 4_s . Pass any horizontal secant plane as U below the points 3, 4, it cuts the circumference L from the surface. The shadow of L on H is L_s , which intersects the shadow of the line of shade at g_{s2} and j'_{s2} ; a ray through each of these points intersects L at g_{s1}

and j_{s1} , hence these points are points of the required shadow. The shadow thus determined is J_{s2} , only part of which is visible as shown. The shadow on H is J_{s1} limited by C_{s1} and A_s which is the shadow of the base A (see below.)

Fourth: The shadows of the element Z and W are Z_s and W_s respectively, and of the upper base A is A_s , as shown. As the lower base D is in H , it is its own shadow.

The complete shadow both on the surface and the coördinate planes is now determined. The visible shadow on the coördinate planes is bounded by the lines $Z_s A_s J_{s1} C_{s1} Y_s B_{s1} B_{s2}$, thence along the V contour of the surface to $G. L.$, thence along $G. L.$ to t_s , thence $C_{s1} J_{s1} A_s W_s A^H$. The visible shadow on the surface is, in V projection, bounded by the lines Z^v , part of A^v , J_{s2}^v , 4^v , 5^v , C_{s3}^v , thence along the V contour of the surface to t^v , $C^v Y^v$, thence along B^v to r^v , $r^v u^v$, thence along the V contour of the surface to $G. L.$, thence along $G. L.$ to t_s . In H projection the visible shadow of the surface is bounded by the lines $J_{s2}^H C^H J_{s2}^H A^H$.

47. Problem XVII. *To find the shadow on the surface of a triangular-threaded screw. Axis vertical. Plate IV, Fig. 22.*

Analysis. The surfaces of a triangular-threaded screw are warped helicoids, and the outside and the root of the thread are helices.

Planes tangent to a warped helicoid at *points on the same helix* make equal angles with the axis and equal angles with H . (See Des. Geom.) If a series of meridian planes of the helicoid be passed perpendicular to these tangent planes, the lines cut from the tangent planes make a constant angle with H equal to the angle which the tangent planes make with H ; and with the elements passing through the points of tangency a constant angle, since the relative positions are the same in all cases. Hence the angle between the H projection of any

line of intersection of the meridian and tangent planes and the H projection of the corresponding element is constant. If now a plane of rays tangent at some point of the given helix be determined, the element containing the point of tangency and hence the point of tangency may be determined. Having determined the line of shade, construct the line of shadow by individual rays.

Construction. Let A be an element of one of the helicoidal surfaces of the screw and C the helix on which a point of the line of shade is to be determined.

The element A pierces H at a , and a tangent line to the helix C at b (the intersection of A and C) pierces H at c^a , hence HT is the horizontal trace of a tangent plane. HU is the H trace of a meridian plane perpendicular to the tangent plane T , hence the angle xoy is the constant angle made by the H projection of the intersection of the two planes with the H projection of the element passing through the point of tangency. The angle which D , the line of intersection of the planes T and U , makes with H is equal to the angle which any tangent plane to the helicoid at a point of the helix C makes with H . Revolve D about the axis X ; a cone is generated with vertex at (d^v, o) and base Q in H . A ray M through the vertex of the cone pierces H at e ; hence HP , through e and tangent to the base, is the H trace of a plane of rays tangent to the cone, and hence is parallel to the H trace of a plane of rays tangent to the helicoid at some point of C .

The H trace of a meridian plane perpendicular to the plane P is of ; hence making the angle $fom^a = \text{angle } xoy$ we have the element which passes through the required point of tangency. This element intersects the helix C at m ; hence m is the point of tangency and therefore a point of the line of shade.

A point of the line of shade on the helix B is determined in the same manner, thus: HQ is the H trace of a tangent plane through the element F on which is the point u of B , and oi is the H trace of a perpendicular meridian plane; hence $io u^H$ is the constant angle. The line G is the intersection of the meridian and tangent planes and pierces H at i ; hence C^H is the base* and j^V , o the vertex of a cone generated by rotating G about the axis, and HR is the H trace of a plane of rays tangent to the cone.

The line ok is the H trace of a meridian plane perpendicular to the plane R ; hence laying off the angle $kog^H = io u^H$ we have the element which intersects the helix B at g , thus determining another point of the line of shade.

To determine an intermediate point of the line of shade: Let K^H be the H projection of an intermediate helix. For convenience pass an auxiliary horizontal plane Z intersecting the helix at w . The intersection with the plane Z of a tangent plane to the helicoid, through the element A at some point of the helix K , is L and of a perpendicular meridian plane is oz ; hence soy is the constant angle. The intersection of the meridian and tangent planes is E , which revolved about the axis generates a cone with base T on the auxiliary plane Z . U is the intersection with Z of a plane of rays tangent to the cone, and or of the perpendicular meridian plane; hence laying off the angle $rot^H = \text{angle } soy$ the element containing the required point t is determined. The V projection of t is t^V on the V projection of the element (not shown). The line of shade on the lower thread is gtm and part of the helix B on the right of g .

As the several threads of the screw are alike in all respects,

* Almost coincides with C^H .

$g^H t^H m^H$ is the H projection of all the lines of shade on the front helicoids, hence $n p q$ is the line of shade of the second thread. The *visible* line of shade is composed of the lines $m t g$, $n p q$ and part of the helices on the right of g and n .

The Shadow. The shadows of $n p q$ and N on H are q_{s2} p_{s2} , etc. (this line is on the right of the border, and hence is not shown). The shadow of any element P of the helicoid Z is P_s . This shadow intersects the shadow of the line of shade at p_{s2} ; hence passing a ray through p_{s2} the shadow p_{s1} on the element P is determined. (See § 30.) In this manner the shadow on V projection $q^V p_{s1}^V n_s l_s$ was determined. The shadow on the lower thread is of exactly the same form. The H projections of the shadows and the shadows on H are not shown.

Problems.

1. Find the shadow of an oblique line on an oblique plane.
2. Find the shadow of a line parallel to $G. L.$ on an oblique plane.
3. Find the shadow of a line situated in a profile plane on the coördinate planes.
4. Find the shadow of a pyramid, base on H , on the coördinate planes.
5. Find the shadow of an inverted pyramid, base parallel to H , on the coördinate planes.
6. Find the shadow of a pyramid, base parallel to H , on the coördinate planes.
7. Given a square pyramid on a square pedestal, find the complete visible shadow.
8. Given a square pyramid on a cylindrical pedestal, find the complete visible shadow.
9. Find the shadow of any parallelopiped on the coördinate planes.

10. Find the shadow of a hollow hexagonal prism on the interior surface and on the coördinate planes.
11. Find the shadow of a vertical cross.
12. Given a vertical cross on a cylindrical pedestal, find the complete visible shadow.
13. Given a square column surmounted by a square abacus, find the complete visible shadow.
14. Given a hexagonal column surmounted by a square abacus, find the complete visible shadow.
15. Find the complete shadow of a rectangular box with its lid partly raised.
16. Given a book-case containing three shelves, find the complete shadow.
17. Given a square-topped table with four square legs, find the complete shadow.
18. Find the complete shadow of any building having the plan and elevation.
19. Given a hollow cylinder, axis parallel to $G. L.$, find the complete visible shadow.
20. Given a hollow cylinder, axis perpendicular to H or V , find the complete visible shadow.
21. Find the shadow of a cone, axis perpendicular to V , on the coördinate planes.
22. Find the shadow of any oblique cone on the coördinate planes.
23. Given a hollow cone, axis parallel to $G. L.$, base on left, find the complete shadow.
24. Given a vertical cylinder surmounted by a square abacus, find the complete visible shadow.
25. Given a cylindrical column surmounted by a cylindrical abacus, find the complete visible shadow.

26. Given a cylindrical column surmounted by a hexagonal abacus, find the complete visible shadow.

27. Find the shadow of an ellipsoid of revolution, axis perpendicular to H or V .

28. Find the shadow of the frustum of a cone.

29. Find the complete shadow of the frustum of a hollow cone.

30. Given a small vertical cylinder intersecting a large horizontal cylinder, find the complete visible shadow.

31. Find the complete visible shadow of a groined arch.

32. Find the complete visible shadow of a cylindrical niche capped by a quarter sphere.

33. Find the shadow on the interior of a recess of any shape.

PERSPECTIVE.

CHAPTER IV.

GENERAL PRINCIPLES, DEFINITIONS AND CONVENTIONS.

48. Perspective in Nature. The phenomena of perspective in nature are so familiar that they scarcely require description. Stand at one end of a long straight street and look towards the other end; the rows of trees on the sides, the fronts of the buildings, the sidewalks and curb-stones, *seem* to approach each other and grow gradually smaller as they recede from the spectator. Straight railroad tracks make an excellent example of perspective, especially if they be several miles long. If you stand on one rail and look along the tracks all the rails will apparently meet it at a distant point. At the sea shore or on shipboard the sky and water seem to meet at the horizon—the effect is the same as if the sky and water were two vast horizontal planes. It is this phenomenon or effect of distance which the draftsman tries to reproduce in a perspective drawing.

49. Deductions. Examples of natural perspective might be multiplied indefinitely, but all teach the same facts, *i. e.*, that any series of parallel lines if sufficiently produced apparently approach each other and meet in a point, and that any series of parallel planes if sufficiently extended apparently approach each other and meet in a line; also, that if a line be

passed through the eye parallel to any series of parallel lines, it will pass through their meeting point, and that if a plane be passed through the eye parallel to any series of parallel planes, it will pass through their meeting line.

As lines or planes may be extended in two directions an infinite distance from any given point, there are two meeting points or lines 180° removed from each other for each series, but as the spectator can only look in one direction at a time it is not necessary to consider the second case. In the case of parallel planes the meeting line would become the circumference of a circle because planes may be infinitely extended in all directions.

50. Methods. The method of perspective in general use by draftsmen is called *One-plane Perspective*, and differs in many essentials from that used by artists, which is called *Cylinder* or *Multiplane Perspective*.

The drawing produced by the *One-plane* method is the same as that which would be made by placing a sheet of transparent paper between the object to be represented and the spectator, and covering or concealing every line which is seen on the object by a line on the paper. NOTE: For purposes of representation the paper might just as well be placed behind the object as in front of it, although not usually so considered. In the first case the drawing is smaller than the object, in the latter case larger.

In *Multi-plane Perspective* a transparent cylindrical surface is supposed to be placed between the spectator, whose eye is in the axis of the cylinder, and the object; lines are then drawn covering or concealing those on the object, and the cylinder is developed. This method is essentially that used by artists, but the difficulty of considering it geometrically is great and the practical application tiresome.

51. Relation to Geometry. Suppose a plane placed between a given point and an object, and the object projected on the plane by means of lines drawn from the point to the object; the drawing obtained will be a duplicate of that made by the process described in § 50. In this case the natural method has been considered geometrically. The lines from the point to the object, *i. e.*, the projecting lines, form a cone because they radiate from a point, hence the object is conically projected. If the object to be projected be a point, the projecting cone becomes a line; if the object be a line, the projecting cone becomes a plane, etc.

52. Definition. *One-plane *Perspective is Conical projection.*

NOTE: If the student will remember that *One-plane Perspective* means *projection* from a fixed center upon a plane, much of the difficulty sometimes found with the subject may be avoided.

In Descriptive Geometry the projecting lines are parallel, in Perspective the projecting lines radiate from a point.

53. Picture-Plane. The *Picture Plane* (also called *Perspective Plane* and *Plane of Projection*) is the plane upon which the object is projected or the drawing is made. It is usually assumed vertical.

54. Perspective. The conical projection of an object on the picture plane is called the *Perspective* of the object.

Any point or line in the picture plane may be considered to be the *Perspective* of some point or line in space.

55. Station Point. The *Station Point* (also called the *Point of Sight*) is the point from which the object is projected on the picture plane. It is the vertex of the projecting cone of the object. This point may, in general, be placed anywhere within a finite distance of the picture plane, but the position

* Also called *Linear Perspective*.

once selected *must* remain fixed for any one drawing. In order that a perspective drawing may appear natural or give one the correct impression of the object represented, the eye of the spectator must be placed at the station point, otherwise the drawing will appear distorted.

56. Center of the Picture. The *Center of the Picture* is the *orthographic projection* of the station point on the picture plane.

57. Axis. The *Axis* of the picture is the perpendicular projecting the station point on the picture plane.

58. System. Any series of parallel lines is called a *System* of lines; any series of parallel planes is called a *System* of planes.

59. Axis of a System. The *Axis of a System* of planes is any line perpendicular to the system.

60. Vanishing Point. Given a *System* of lines and a point: If a plane be passed through each line and the point the common intersection of the planes is parallel to the *System* of lines. Given a *System* of lines, the *Station Point* and the *Picture Plane*: If a plane be passed through each line of the system and the station point, the intersection of each plane with the picture plane is the *Perspective* of the line through which the plane is passed (§ 54). But these planes have a common intersection which pierces the picture plane in a point, and the intersection of each plane with the picture plane passes through this point; hence the *Perspective* of each line passes through this point. The point of intersection of the perspectives of a system of lines is called the *Vanishing Point* of that system or of any line of that system.

The *Vanishing Point* of a line may also be defined as the perspective (§ 54) of that point on the line which is at an infinite distance from the station point. Since perspective is conical projection and parallel lines intersect only at infinity,

the projection (perspective) of this point of intersection on the picture plane must be determined by a line through the station point parallel to the given line.

61. Determination of Vanishing Points. Since the common intersection of the projecting (conical projection) planes of a system of lines is parallel to the system and passes through its vanishing point (§ 60), for any system of lines the vanishing point is determined by the intersection with the picture plane of a line of the system through the station point.

62. Vanishing Points of Systems of Lines Parallel to the Picture Plane. The perspectives of a system of lines parallel to the picture plane are parallel to each other and to the system, for the common intersection of their projecting planes is parallel to the picture plane and therefore intersects it only at infinity. This fact limits but does not violate the preceding rule (§ 61).

63. Vanishing Trace. The *Vanishing Trace* of a plane is the locus of the vanishing points of all lines in the plane.

64. Horizon. The *Horizon* is the vanishing trace of horizontal planes or the locus of the vanishing points of horizontal lines. It is the horizontal line in the picture plane which passes through the center of the picture (§ 56).

65. Determination of Vanishing Traces. For any system of planes the vanishing trace is determined by the intersection with the picture plane of a plane of the system through the station point. (See § 61.)

66. Relative Position of Vanishing Points and Vanishing Traces. From §§ 61 and 63 it follows that the vanishing point of a line is in the vanishing trace of *any* plane which contains that line. Also, that if a line be the intersection of two planes its vanishing point is the intersection of the vanishing traces of the two planes.

67. Initial Point. The *Initial Point* of a line is the point in which the line pierces the picture plane. As the picture plane is usually assumed vertical, *initial point* is only another name for *vertical trace*. Throughout this book the initial point of a line is the vertical trace of the line, and the initial line (§ 68) of a plane is the vertical trace of the plane. As the initial point of a line is in the picture plane it is one point in the *perspective* of the line.

68. Initial Line. The *Initial Line* of a plane is the line in which the plane intersects the picture plane. *The initial line of a plane is parallel to the vanishing trace of the plane* (§§ 63 and 65).

69. The *Initial Point* of a line determines the *position* of the perspective of the line. The *Vanishing Point* of a line determines the *direction* of the perspective of the line.

70. Principal Lines of an Object. The *Principal Lines* of an object are the systems of lines which most frequently appear in the representation of the object.

Take, for example, a building such as a factory: the principal lines shown in the drawing belong at most to but three systems. First: On the front of the building, the *horizontal lines*, such as the courses of brick, water-tables, window-sills, etc. Second: On the side of the building, the *horizontal lines* corresponding to those on the front. Third: On the front and side, the *vertical lines*, such as the edges of the building, window-frames, etc.

Hence in order to make a perspective drawing of an object which consists essentially of systems of right lines, it is necessary in general to determine but three vanishing points.

71. Position of Object to be Represented. Objects which have three (or less) systems of principal lines may be placed, with reference to the picture plane, in three general

positions. First: With the systems parallel to, and perpendicular to, the picture plane. Second: With one system parallel to, and two systems oblique to, the picture plane. Third: With the three systems oblique to the picture plane.

72. Parallel or One-Point Perspective. The representation of an object situated as in Case I, § 71, is called *Parallel* or *One-Point Perspective*, because but one vanishing point is required to make the drawing.

73. Angular or Two-Point Perspective. The representation of an object situated as in Case II, § 71, is called *Angular* or *Two-Point Perspective*, because two vanishing points are required to make the drawing.

74. Oblique or Three-Point Perspective. The representation of an object situated as in Case III, § 71, is called *Oblique* or *Three-Point Perspective*, because three vanishing points are required to make the drawing.

75. Notation.

- (1) The *position* of the *station point* is denoted by e .
- (2) The *center of the picture* is denoted by e^v (see § 56).
- (3) In general, when the projections of an object are attached to the drawing, the *perspective* of a *point* is denoted by the letter of the point with the exponent $'$; *e. g.*, a' , b' , etc.
- (4) The *perspectives* of a *system of lines* are denoted by the letter of the system with the exponents $^1, ^2, ^3$, etc.; *e. g.*, R^1 , R^2 , etc.
- (5) The *vanishing point* of a system of lines is denoted by a capital V with an exponent which is the letter of the system, *e. g.*, V^R , V^P , etc.
- (6) The *vanishing trace* of a system of planes is denoted by a capital T succeeded by the letters of any two systems of lines in the planes, *e. g.*, T R P., T M N, etc.

(7) The *initial point* of a line is denoted by a capital I with an exponent which is the letter of the line; *e. g.*, I^R , I^B , etc.

(8) The *initial line* of a plane is denoted by a capital I succeeded by the letters of any two lines in the plane, or of any line in the plane, or by the letter of the plane; *e. g.*, $I R B$, $I M$, $I E$, etc.

(9) A *point of distance* for a system of lines is denoted by a capital D with an exponent, which is the letter of the system; *e. g.*, D^R , D^M , etc.

(10) A *point of proportional distance* for any line is denoted by a capital P with an exponent, which is the letter of the line; *e. g.*, P^M , P^R , etc.

76. Perspective is Descriptive Geometry. Plate V, Fig. 23. This figure is the Descriptive Geometry representation of three lines, Q , W , and U , and of a plane P , passed through the intersecting lines Q and W .

To find the vanishing points of the lines Q , W , U , and the vanishing trace of the plane P . Plate V, Fig. 23a. Let the picture plane be the V coördinate plane of Fig. 23, and also use the H coördinate plane of the same figure. Let e^H , e^V be the orthographic projections of the station point (§ 55), which is about one and one-half inches in front of the picture plane. e^V is the center of the picture (§ 56). Through the station point pass a line parallel to Q : it pierces the picture plane at V^Q ; hence V^Q is the vanishing point (§§ 60 and 61) of Q and of all lines parallel to Q , *i. e.*, of the system Q . In the same manner the vanishing points V^U and V^W of the lines U and W respectively, were determined.

The line $e^V V^Q$ is the horizon, since U and Q are horizontal (§ 64), and $T W Q$, passing through V^W and V^Q , is the vanishing trace of the plane P and of all planes parallel to P , *i. e.*, of the system P , since it contains the vanishing points of two lines of

that plane (§§ 65 and 66). TWQ is parallel to VP (Fig. 23).

It is thus evident that the determination of a vanishing point is the determination of the V trace of a line passed through a given point, and that the determination of the vanishing trace of a plane is the determination of the V trace of a plane which passes through two given lines intersecting at a given point.

77. To find the perspective of a rectangular prism by direct projection. Plate V, Fig. 26.

Analysis. Since perspective is conical projection or projection from a given point upon a plane (§§ 51, 52, 54), it follows that if a line be drawn from the given point (station point, § 55) to each point of the figure, that the intersections of these lines with the plane (picture plane, § 53) give the perspective of the figure. This is the same as finding the V traces of a number of lines which radiate from a point. In this method both projections (H and V) of the object are required, and are usually placed so that the object is in either the second or third dihedral angle.

A much simpler solution than the above, which may be called direct projection, will be explained in the next chapter.

Construction. Let $abcdikgf a$ be a rectangular prism situated in the second dihedral angle. The top and bottom faces are parallel to H and the other four faces are vertical planes inclined at angles of 60° and 30° to V as shown. The station point (§ 55) is e and the picture plane (§ 53) is V . From e draw a line to a : it intersects the picture plane (V) at a' ; hence a' is the *perspective* of the point a . From e draw a line to k : it intersects the picture plane (V) at k' ; hence k' is the *prospective* of k . In like manner draw lines to all the corners of the prism and find their intersections with the picture plane (V).

These intersections or perspectives joined in the proper manner give the perspective $a^1 b^1 d^1 c^1 i^1 k^1 g^1 f^1 a^1$ of the prism.

Note the following facts : The perspective lines $a^1 f^1$, $c^1 i^1$, $d^1 k^1$, and $b^1 g^1$ converge and meet at V^N , which is the vanishing point of these lines (§ 60). This is as it should be, because the corresponding lines on the prism are parallel, and hence must have a common vanishing point. Through e (station point) draw a line parallel to one of these lines on the prism, e. g., $a f$: it intersects the picture plane at V^N , which is the same vanishing point as before. Hence we reach the same result whether we begin by determining the vanishing point of a system of lines (§ 61), or whether we find the vanishing point from the perspectives of the lines. Similarly the vanishing point of the lines inclined to the right is V^K (not shown, about one inch to right of border line on $V^N e^v$). The perspective lines $a^1 b^1$, $c^1 d^1$, $i^1 k^1$, and $f^1 g^1$ are parallel, which is correct, because the corresponding lines on the prism are parallel to the picture plane (V); hence their vanishing point is at infinity, for if a line be drawn through the station point (e) parallel to these lines it will intersect the picture plane only at infinity. The systems $a f$, $c i$, etc., and $a c$, $b d$, etc., are horizontal, hence the line $V^N e^v$ is the horizon (§ 64) or the vanishing trace (§ 63) of the planes $a f i c$ and $b g k d$, because it contains the vanishing points (V^K and V^N) of two or more lines in each plane. This is also true from the fact that the two planes $a f i c$ and $b g k d$ are horizontal; hence if a plane be passed through the station point parallel to these planes it will intersect the picture plane in the line $V^N e^v$, or the horizon.

As the planes $a b g f$ and $c d k i$ are *vertical* their vanishing trace must be perpendicular to $G. L.$ And as V^N is the vanishing point of a line in each plane, $V^N z$ is the vanishing trace of

these planes. This fact might also have been established by finding the vanishing point of any other line in either plane, *e. g.*, one of the diagonals of either of the two faces.

The V trace of the line af , or the point in which the line pierces the picture plane, is q^v ; hence q^v is the *initial point* (§ 67) of the line af . The perspective of this line, $a'f'$, passes through q^v , because q^v is a point in the line and also a point in the picture plane, in other words, q^v and the *perspective* of q^v coincide.

Similarly m^v is the initial point of fi and the perspective $f'i'$ passes through it, also n^v is the initial point of gk and the perspective $g'k'$ passes through it. Since m^v and n^v are the initial points (V traces) of two lines in the plane $fgki$, the line $m^v n^v$ must be the initial line (§ 68) or V trace of the plane $fgki$. Also $m^v q^v$ is the initial line of the plane $afic$. The line $m^v q^v$ is horizontal, which is correct, since the vanishing trace $V^N e^v$ is horizontal, and hence parallel to $m^v q^v$ (§ 69). We thus establish the fact that the vanishing trace and initial line of a plane are parallel, which must be true because the plane which determines the vanishing trace is parallel to the plane which has the initial line and the intersections of parallel planes with the same plane are parallel. The line $m^v n^v$ which is the initial line of the plane $fgki$ is also parallel to the vanishing trace (not shown) of the plane. The line $q^v q^v$ is the initial line of the plane $afgb$ because it contains the initial point q^v of a line in the plane and is parallel to $V^N z$ which is the vanishing trace of the plane. We might also have determined this initial line by drawing a line through q^v parallel to ab , since the initial point of ab is at infinity. Note also the fact that the initial point of a line is at the intersection of the initial lines of any two planes which con

tain the line, e.g., m^v , and that the vanishing point of a line is at the intersection of the vanishing traces of any two planes which contain the line ; e.g., V^N is the vanishing point of af and the intersection of $e^v V^N$ and $s V^N$.

For the duplication of this problem by the method of measurement see Prob. I, § 95.

CHAPTER V.

THE MEASUREMENT OF LINES IN PERSPECTIVE.

78. The perspectives of lines which lie in the picture plane coincide with the lines, and hence are equal to them in length. In general, however, a line and its perspective are unequal in length.

If by any means we may transfer distances from the perspectives of lines which are actually in the picture plane to the perspectives of other lines which are not in the picture plane, a method of measuring or laying off distances in perspective is effected.

79. Proposition I. If parallels to the base of a triangle be drawn, the corresponding segments on the two sides are in a constant ratio. (See Plate VII, Fig. 34.)

If the triangle be isosceles the ratio is unity, hence the corresponding segments on the two sides are equal. (See Plate VII, Fig. 35).

80. Proposition II. A line drawn through the intersection of the diagonals of a parallelogram parallel to two of its sides bisects the other two sides and the parallelogram. (See Plate VII, Fig. 36.) This method of subdivision may be continued as shown in the figure.

81. Proposition III. If one side of a parallelogram be divided in any way at one end, equal divisions may be laid off at the other end by means of two diagonals. (See Plate VII, Fig. 37.)

82. To duplicate Proposition I in perspective. Plate V, Fig. 24. *Isosceles triangle.*

Analysis and Construction. Let axx be an *isosceles* triangle situated in the third dihedral angle and having one of its equal sides, ax , in V (or picture plane.) Assume $G. L.$ to be the horizon, e the station point, and c^v the center of the picture (§ 56).

Find the perspective of the triangle. The vanishing point of ax (J) is V^J found by passing a line through the station point e parallel to J (see § 77). Since a^v is in the picture plane it is the initial point of J , and hence one point in the perspective of J ; hence $a^v V^J$ is the indefinite perspective of J . The vanishing point of xx (F) is D^J , found by passing a line through the station point parallel to F , and x^v is the initial point of F ; hence $x^v D^J$ is the indefinite perspective of the line. The lines $a^v V^J$ and $x^v D^J$ intersect at z^1 , which is therefore the perspective of the point z .

Since ax is in the picture plane, $a^v x^v$ is the perspective, or in other words the line is its own perspective. Hence $a^v z^1 x^v$ is the perspective of the triangle axx . But the triangle axx is *isosceles*, hence $a^v x^v = a^v z^1$ in perspective, that is, the perspective length $a^v z^1 =$ the true length $a^v x^v$. Parallels to the base (F) of the triangle vanish at D^J ; hence if we lay off any distance on $a^v x^v$, as for example $a^v c = 1/2''$, and draw $cm D^J$, we will again have an isosceles triangle in which $a^v c = 1/2'' = a^v m$ in perspective. Also lay off any other distance as $a^v d = 1\frac{1}{4}''$, draw dd^J intersecting J^1 at p ; then $a^v d = 1\frac{1}{4}'' = a^v p$ in perspective.

Notice that $T J F$ is the vanishing trace of the plane of the triangle because it contains the vanishing points of two sides of the triangle, and that $a^v x^v$ is the initial line of the plane of the triangle, and also that $T J F$ and $a^v x^v$ are parallel (see §§ 68 and 77).

As the chief requirement is to find a means to lay off dis-

tances on the perspective J^1 , any line in the picture plane other than ax which would complete an isosceles triangle having az for one of its equal sides may be used. Let $ay = ax = az$ be such a line; then we have the isosceles triangle $az y$. The vanishing point of the base yz is D^J ; hence $a^v z^1 y^v$ is the perspective of the triangle $az y$, and we may use the side $a^v y^v$, which is in the picture plane, to lay off distances to be transferred (by parallels to the base) to the perspective side J^1 .

82a. Usually in a problem where measurements are necessary we have the following data (Fig. 24): The vanishing point of the line as V^J , the indefinite perspective of the line as J^1 , the vanishing trace of a plane containing the line as $T J F$, the horizon as $G. L.$ ($G. L.$ is the horizon only for convenience), and the station point e . It is required to transfer measurements to the line J^1 in perspective by means of an isosceles triangle. We know that the line $T J F$ is parallel to the initial line of the plane of the triangle, and hence is parallel to one of the equal sides; we also know that the line joining the station point with the vanishing point V^J of the line is parallel to the line and hence parallel to the other equal side; hence all we are required to find is the base or vanishing point of the base of the triangle having for its equal sides $e V^J$, and part of $T J F$ equal in length to $e V^J$, to form a triangle whose sides are respectively parallel to the triangle in space which has the side J .

Revolve the plane containing $T J F$ and e about $T J F$ (which is the V trace of such plane) into V (picture plane): the station point e revolves to e_1 , and the line $e V^J$ to $e_1 V^J$.

Now lay off $V^J D^J = V^J e_1$, and join $D^J e_1$, and we have an isosceles triangle $V^J D^J e_1$. But D^J is a point in the picture plane, and hence the vanishing point of the base $e D^J$ of the

isosceles triangle $e V^J D^J$. We have thus determined the same point, D^J , for the vanishing point of the base of the isosceles triangle that we had in § 82.

The point D^{J1} or D^J is called a *Point of Distance* (§ 83), and the line $a^v x^v$ or $a^v y^v$ is called a *Line of Measures* (§ 87).

83. Point of Distance. A *Point of Distance* for a given oblique line is the vanishing point of the base of an isosceles triangle whose equal sides are the given line and the initial line of any plane which contains the given line. From § 82a we may derive the following definition: A *Point of Distance* for a given oblique line is the vanishing point of the base of an isosceles triangle whose equal sides are, the line from the station point to the vanishing point of the given line, and the vanishing trace of any plane containing the given line.

84. Since a line may be contained in an infinite number of planes having an infinite number of vanishing traces, it follows that a *Point of Distance* for a line may be any point in the picture plane situated as far from the vanishing point of the line as the vanishing point of the line is from the station point.

85. When the given line is *parallel* to the picture plane its vanishing point is at infinity; hence the equal sides of the isosceles triangle are parallel, and the point of distance may be any point on the vanishing trace of a plane containing the line (see §§ 95 and 97).

86. Point of Half Distance. Plate V, Fig. 24. Suppose the lines revolved, etc., as in § 82a. Bisect $V^J D^J$, and draw $e_1 D^{1J}$; a triangle $e_1 D^{1J} V^J$ is formed, of which the side $V^J D^{1J}$ is one-half $e_1 V^J$. D^{1J} is the vanishing point of the side $e D^{1J}$ of the triangle. Hence if we lay off on $a^v x^v$ distances to *half scale* and draw lines to D^{1J} intersecting J^1 , the same subdivisions are effected as in § 82; *e. g.*, lay off $a^v f = \frac{1}{4}$ inch, draw $f D^{1J}$ intersecting J^1 at m , then $a^v m$ in perspective = $2 a^v f$,

because the sides of the triangle $a^v f m$ are parallel respectively to the sides of the triangle $e D^{1/2} V^J$. The point $D^{1/2}$ is called a *Point of Half Distance*. If one-quarter of $V^J e_1$ were laid off on $V^J D^J$ from V^J , a *Point of Quarter Distance* would be obtained, and the distances on $a x$ would be laid off to quarter scale. This process may, of course, be continued indefinitely, and *Points of Three-Quarter Distance, One Tenth Distance*, etc., obtained.

87. Line of Measures. A *Line of Measures* for a given line is the *initial line* of the plane containing the given line and its point of distance. Hence a line of measures for an oblique line is a line in the picture plane which passes through the *initial point* of the line and is *parallel* to the line joining the vanishing point of the given line with its point of distance (see § 82a).

88. Plane of Measures. A *Plane of Measures* is a plane in which the measurements which are to be transferred to the perspective are made.

Throughout this book the *Plane of Measures* is supposed to coincide with the picture plane.

89. Scale. The scale to which the drawing is made may be anything convenient, such as 1 inch to 10 feet, 1 inch to 1 foot, etc., or even 1 inch to 1 inch, and depends entirely upon the purposes for which the drawing is required. A decimal or chain scale is, however, preferable to any other although *not used* in practice.

90. Proportional Measures. It is often necessary to divide a given line into a certain number of equal or proportional parts irrespective of the actual length of any part. For this purpose a method somewhat shorter than the method of equal measures (§ 82 *et seq.*) is given.

To duplicate Proposition 1 in Perspective. Plate V, Fig. 25. Any triangle. See § 82a.

Analysis and Construction. Let V^J be the vanishing point of a line, and TJ the vanishing trace of any plane passed through the line J . Revolve the plane of TJ and e about TJ as an axis into the picture plane: the point e revolves to e_1 , and the line eV^J to e_1V^J . Draw any line as e_1P^J , thus completing a triangle $e_1V^JP^J$. Revolve the triangle back to its true position, and P^J is the vanishing point of the base eP^J .

Let I^Jb be the perspective of a line of the system J which is to be divided into three (or any number) of equal (or unequal) parts.

The point I^J is the initial point of J^1 , but as far as the solution of the problem is concerned may be anywhere on the perspective line J^1 . Through I^J draw I^Jd parallel to TJ , draw P^Jbd : then the perspective triangle I^Jbd is similar to the triangle eV^JP^J . Divide I^Jd into three equal parts (the method of geometric division is shown), and from the points of division draw pP^J and qP^J intersecting J^1 at m and n respectively: then $I^Jp : pq : \text{etc.}, :: I^Jm : mn : \text{etc.}$, in perspective. The point P^J is called a *Point of Proportional Distance*, and the line I^Jd is called a *Line of Proportional Measures*. Any other point as P^{J1} might have been taken on TJ , and the same subdivisions affected, as shown in the figure. Instead of using the line I^Jd through the initial point of J , we may use any other line parallel to TJ , as mg , and obtain the same result.

91. Point of Proportional Distance. A *Point of Proportional Distance* for an oblique line is the vanishing point of the base of a triangle, the sides of which are the given line, and any line parallel to the vanishing trace of a plane which contains the given line.

92. Line of Proportional Measures. A *Line of Proportional Measures* for an oblique line is a line drawn through *any point* of the given line *parallel* to the vanishing trace of any plane containing the given line.

93. Of any two perspective lines having the same vanishing point, one may be taken as the trace of a plane passing through the other; and if a third line be drawn parallel to the first through any point of the second, any parts taken upon this third line may be transferred to the second in their true proportions by means of a point of proportional distance taken upon the first.*

94. It is evident from the preceding paragraphs that lines which are parallel to the picture plane may be sub-divided directly.

* Modern Perspective, by W. R. Ware.

CHAPTER VI.

PARALLEL, ANGULAR, AND OBLIQUE PERSPECTIVE.

95. Problem I. *To find the perspective of a rectangular prism.* Angular perspective. Plate V, Fig. 27.

Analysis. This problem is a duplication of § 77 (Fig. 26) by the method of measurements described in the last chapter. The prism is supposed to rest on a horizontal plane through $G. L.$ which corresponds to the line $m^v q^v$ of Fig. 26. All dimensions are given in Fig. 26. The principal (and only) lines of the prism are the edges which belong to three systems (§ 70). Four of the edges are horizontal and inclined to the right (K lines), four are horizontal and inclined to the left (N lines), and four are parallel to the picture plane and perpendicular to H (A lines). Hence the drawing is to be made in *angular perspective* (§ 73), and only the two vanishing points of the horizontal edges are required.

Construction. Let $V^N V^K$ be the horizon, assume the station point $2\frac{1}{2}''$ ($= e^H x$, Fig. 26) in front of the picture plane, and let e^v be the center of the picture (§ 56). Consider a horizontal plane passed through $V^N e^v V^K$; then e_1 is the projection of the station point on this plane, and also its true position, and $e_1 e^v = 2\frac{1}{2}''$. This horizontal plane through the horizon is shown in a number of the following problems and is useful in determining vanishing points, etc.

Since K and N are *horizontal*, their vanishing points are on the horizon (§ 64); hence by drawing through e_1 a line K making an angle of 60° with $V^N V^K$ the vanishing point V^K of

the edges inclined to the right is determined, and through e_1 a line N perpendicular to K the vanishing point V^N of the edges inclined to the left is determined. A point of distance for K is D^K , found by laying off $V^K D^K = V^K e_1$ on the horizon (§§ 82-84), and a point of distance for N is D^N , found by laying off $V^N D^N = V^N e_1$.

In practical work it is usually much easier to compute the positions of vanishing points and points of distance than to determine them by geometric methods. In the present problem the angles which the edges make with the picture plane are given; hence $e^v V^K = 2.5 \tan 30^\circ = 1.44''$, $e^v V^N = 2.5 \tan 60^\circ = 4.33''$, $V^N D^N = \frac{2.5}{\sin 30^\circ} = 5.00''$, and $V^K D^K = \frac{2.5}{\sin 60^\circ} = 2.89''$. These distances may be laid off at once.

Since the prism rests on a plane through $G.L.$, this line must be the initial line (§ 68) of the lower face of the prism; and as it is parallel to the horizon which contains the vanishing points and points of distance of the horizontal lines of this face, it is also a line of measures (§ 87) for this face. The point a (Fig. 26) is $1\frac{5}{8}''$ to the left of the station point and $3/4''$ behind the picture plane. Hence lay off $j I^P$ (Fig. 27) $= 1\frac{5}{8}''$. The perspective of the point a is on a perpendicular to the picture plane through I^P , and as all perpendiculars vanish at e^v (V^P), $I^P e^v$ must contain the perspective of the point. A point of distance for $I^P e^v$ is $V^{45^\circ L}$, found by laying off $e^v V^{45^\circ L} = e_1 e^v$, since the perpendicular is in the plane of the (lower) face. Lay off $I^P j = 3/4''$ and draw $j V^{45^\circ L}$ intersecting $I^P e^v$ at a' ; we thus have an isosceles triangle formed in which $I^P j = I^P a'$ in perspective; hence a' is the perspective of the point required (§ 82). Draw the indefinite lines K' , N' , and A' for the indefinite perspectives of the edges of the prism which intersect at a (Fig. 26).

We know that the initial points of the lower edges are on $G.L.$, because $G.L.$ is the initial line of the lower face; also that this line is a line of measures and we have determined the points of distances for N^1 and K^1 . The initial point of N^1 is I^{N^1} , found by producing N^1 to $G.L.$ Draw $D^N a^1 p$ intersecting N^1 at a^1 ; then $I^{N^1} a^1 p$ is an isosceles triangle in which $I^{N^1} p = I^{N^1} a^1$ in perspective. Lay off $p m = 1\frac{1}{4}'' (= a f, \text{ Fig. 26})$ and draw $m D^N$ intersecting N^1 at f^1 ; then $p m = a^1 f^1$ in perspective, being corresponding segments on the equal sides of an isosceles triangle (§§ 79 and 82), and the perspective of the lower left edge is determined. It was of course unnecessary to find the initial point I^{N^1} except to show how the isosceles triangle was formed as we wished to lay off a distance from a^1 . To lay off the length of K^1 draw $D^K a^1 n$, lay off $n q = 1/2'' (= a c, \text{ Fig. 26})$, draw $q D^K$ intersecting K^1 at c^1 , then $n q = a^1 c^1$ in perspective. We may now draw $N^2 K^2$, which intersect at i^1 , and thus complete the perspective of the lower face. Draw the indefinite lines $f^1 g^1, i^1 k^1$, and $c^1 d^1$ perpendicular to $G.L.$ (*i. e.*, parallel to the corresponding lines of Fig. 26) for the perspectives of the edges which are parallel to the picture plane. To lay off $f^1 g^1$: The point m is the initial point of the line $m f^1 D^N$ (see before), and this line passes through the point f^1 of $f^1 g^1$. The initial point of $f^1 g^1$ is at infinity; hence $m r$ parallel to $f^1 g^1$ is the initial line of a plane containing $f^1 g^1$, and also a line of measures for $f g^1$. Lay off $m r = 3/4'' (= f g, \text{ Fig. 26})$, draw $D^N r$ intersecting $f^1 g^1$ at g^1 , then $m r = f^1 g^1$ in perspective. This is true, because $m r$ and $f^1 g^1$ are actually parallel, and $m D^N$ and $r D^N$ are parallel in perspective.

NOTE: The measurement of a line parallel to the picture is effected by laying off distances on the initial line of any plane containing the line (§ 85), and transferring them by means of

parallels to the perspective of the line. In the case of lines parallel to the picture plane the initial line is *parallel* to the perspective line.

Similarly we may lay off $c^1 d^1$ as follows: I^{N^2} is the initial point of N^2 , which passes through the point c^1 of $c^1 d^1$; hence $I^{N^2} I^{N^4}$ is a line of measures for $c^1 d^1$, to which it is parallel. Lay off $I^{N^2} I^{N^4} = 3/4'' (= c d, \text{Fig. 26})$, draw $I^{N^4} V^N$ intersecting $c^1 d^1$ at d^2 ; then $c^1 d^2$ is the perspective required. Draw N^3 ($e^1 b^1$) intersecting A^1 at b^1 , also K^3 ($b^1 d^1$), N^4 and K^4 thus completing the perspective of the prism, which is a duplicate of Fig. 26.

Instead of erecting the perpendiculars to $G. L.$, $a^1 b^1$, $f^1 g^1$, $i^1 k^1$, and $c^1 d^1$, and laying off the height of the prism upon one of them, we may find the perspective of the upper face directly as follows: Since the prism is $3/4''$ high and the plane of the upper face is horizontal, lay off $m r = 3/4''$ and draw $r I^{N^4}$ parallel to $G. L.$; then $r I^{N^4}$ is the initial line of the upper face, and also its line of measures, and we may duplicate in this plane the work of finding the perspective of the lower face.

After the perspective of the upper face is determined by this method join the corresponding points in the two faces to complete the perspective.

96. Object at 45° . When an object is so placed that a number of its principal lines (§ 70) are horizontal and inclined at an angle of 45° to the picture plane, the determination of the vanishing points and points of distance is much simplified, as these points are symmetrically placed with reference to the center of the picture.

The vanishing points of horizontal lines making angles of 45° with the picture plane are on the horizon at a distance from the center of the picture equal to the length of the axis (§ 57); hence, if the center of the picture and one of these

vanishing points be given or assumed, the position of the station point is known.

Plate VI, Fig. 28. The horizon is the horizontal line passing through e^v , the center of the picture, and $V^{45^\circ R}$ and $V^{45^\circ L}$ are the vanishing points of horizontal lines making angles of 45° to the right and left. These points are usually denoted $V^{45^\circ R}$ and $V^{45^\circ L}$, or simply V^{45° . $V^{45^\circ R} e^v = V^{45^\circ L} e^v = \text{length of axis} = 3 \text{ inches (in this case)}$; hence the station point is 3 inches in front of the picture plane. $T W J$ and $T F O$ are the vanishing traces of diagonal planes (§ 44), and $T Y Z$ is the vanishing trace of profile planes. Lines in profile planes and making angles of 45° with the picture plane vanish at V^Y and V^Z ; hence $V^{45^\circ R} e^v = e^v V^Z = e^v V^Y = V^{45^\circ L} e^v$, and $V^{45^\circ} V^Y = V^{45^\circ} V^Z = \text{etc.} = \text{the distance that the station point is from } V^{45^\circ R}, \text{ or } V^{45^\circ L} = \text{the distance that a point of distance (§§ 83 and 84) for } R, L, Z, \text{ or } Y \text{ lines is from } V^{45^\circ R}, V^{45^\circ L}, V^Z, \text{ or } V^Y \text{ respectively. With } V^{45^\circ R} \text{ as a center and radius} = V^{45^\circ R} V^Y \text{ describe a circle (only an arc is shown); this circle is the locus of the points of distance for the system of lines vanishing at } V^{45^\circ R}.$

Similarly with $V^{45^\circ L}$ as a center and radius $= V^{45^\circ} V^Y$ describe a circle which is the locus of the points of distance for the system vanishing at $V^{45^\circ L}$. The symmetry of the 45° position is thus shown.

The vanishing point of any line in a diagonal plane may be readily determined if the angle which it makes with a horizontal plane be known, as follows: The line if inclined to the right vanishes in $T W J$, if inclined to the left it vanishes in $T F O$ (§ 63).

If the station point be revolved about $T W J$ into the picture plane, $D^{45^\circ R}$ is its revolved position, since $V^{45^\circ R} D^{45^\circ R} = V^{45^\circ R} e$; hence through $D^{45^\circ R}$ draw *any line*, as $D^{45^\circ R} V^J$, making

the given angle (30° in this case) with the horizon. V^J , the intersection of $D^{45^\circ R} V^J$ and $T W J$, is the vanishing point required. Similarly V^W , V^O , and V^F are the vanishing points of lines in diagonal planes inclined down to the right and up and down to the left respectively, and making angles of 30° with a horizontal plane. We might at once have laid off the distances $V^{45^\circ R} V^W = V^{45^\circ R} V^J = V^{45^\circ L} V^O = V^{45^\circ L} V^F = 3 \sqrt{2} \tan 30^\circ = 2.45''$. $V^J D^{45^\circ R} = V^J e$; hence, if with V^J as a center and radius $V^J D^{45^\circ R}$ we describe a circle, the locus of the points of distance for the system vanishing at V^J is obtained. Notice that this circle passes through V^W . Why?

$T F R$, $T L W$, $T O R$, and $T L J$ are the vanishing traces of planes containing horizontal lines inclined at angles of 45° to the picture plane, and lines in diagonal planes inclined at angles of 30° to the horizontal plane.

Another advantage in placing an object at 45° is that in this position the station point is at a maximum distance from the picture plane, and the drawing is concentrated within narrower limits; *e. g.*, Plate V, Fig. 27, the object is placed at 30° and 60° , and the station point is at the distance $e_1 e^v$ from the picture plane; if now the station point be moved to e_2 , its distance from the picture plane is $e_2 D^K > e_1 e^v$, and the vanishing points, V^K and V^N , used for the 30° , 60° position, are now used for the 45° position, since they are equally distant from e_2 , and $V^N e_2 V^K$ is a right angle, being inscribed in a semicircle.

97. Different Ways of Measuring the Same Line. Plate VI, Fig. 28.

Analysis and Construction. Let J be the indefinite perspective of a line which vanishes at V^J , and let R be the perspective of its horizontal projection on a plane through I^R ; then I^R is the initial point of R , and $I W J$ perpendicular to the hori-

zon (*i. e.*, parallel to $T W J$) is the initial line of a plane containing R and J .

Note the fact that the vanishing point of a line and the vanishing point of its horizontal projection are in the same perpendicular to the horizon.

Produce J until it intersects $I W J$ at I^J , which is the initial point of J . It is required to lay off $2.5''$ on J from the point p . Since V^J is the vanishing point of the line, $T W J$, $T L J$, and $T Q J$, all passing through V^J , may be considered the vanishing traces of planes containing the line, and V^W , D^J , and V^Q on the "locus of D^J " are the corresponding points of distance (see § 96). (a) We have already found the line of measures $I W J$ parallel to $T W J$; hence draw $V^W p i$ intersecting $I W J$ at i , from i lay off $i k = 2.5''$, draw $k V^W$ intersecting J at q ; then $p q$ in perspective $= 2.5''$. (b) Through I^J draw $I L J$ parallel to $T L J$ for a line of measures, draw $D^J p f$, from f lay off $f g = 2.5''$ and draw $g q D^J$; then $p q$ in perspective $= 2.5''$ as before. (c) Through I^J draw $I Q J$ parallel to $T Q J$, draw $V^Q p r$, from r lay off $r t = 2.5''$ and draw $t q V^Q$; then $p q$ in perspective $= 2.5''$. Three ways of obtaining the same result have thus been shown.

The measurement of the line L , which is in a horizontal plane through I^L , is also shown. V^U is a point of distance, being on "locus of D^L " (see § 96), and $I^L z$, parallel to $T L U$ and passing through I^L , the initial point of the line, is a line of measures; then $w x$ in perspective $= y z$.

Let A be the perspective of a line parallel to the picture plane and B the perspective of its projection on a horizontal plane, of which $G. L.$ is the initial line. It is required to lay off a certain distance on A from n to the left.

Suppose we measure the line by parallels (§ 85) situated in profile planes (compare with § 95). [NOTE: We make this as-

sumption only for convenience in drawing; any other assumed position of the parallels would have answered the purpose.] As the vanishing trace of profile planes is $T Y Z$ (a line through e^v perpendicular to the horizon), the point of distance or vanishing point of the parallels to be used must be on $T Y Z$. Through n, l pass a profile plane, its initial line is $I N M$; hence a line of measures for A must have one extremity in $I N M$, because this line is the initial line of a profile plane through one extremity of A , and hence must contain the parallel which passes through n . Assume V^N , or any point on $T Y Z$, as the vanishing point of the parallels, draw $V^N n d$ intersecting $I N M$ at d ; then d is the initial point of this line. Through d draw $d j$ parallel to A for a line of measures, lay off the required distance $d j$ and draw $j m V^N$; then $d j = n m$ in perspective. The same measurement is found by using V^M .

98. Problem II. *To find the perspective of a cube.* Parallel and angular perspective. Plate VII, Figs. 30–33 inc.

(a) *Analysis.* The four equal cubes shown are supposed to be placed in a room; two are on or near the floor and two are fastened to the ceiling. The length of the edge of each cube is 4 feet and the room is 14 feet high. The scale of the drawing is 1 inch = 2 feet. The station point is 7 feet in front of the center of one wall which is taken as the picture, plane and behind which, in the room, the cubes are placed. The lower left cube (Fig. 30) has one face in the floor, and one face parallel to, and 1 foot behind, the wall. The lower right cube (Fig. 31) has one edge in the floor and one edge in the wall, and the face passing through these two edges is inclined 45° to the floor. The upper cubes (Figs. 32 and 33) are similarly situated in regard to the ceiling and wall; each has one face in the ceiling, one edge parallel to, and 1.5 feet behind, the wall, and the faces passing through this edge are inclined 45° to the wall.

Hence Fig. 30 is in parallel perspective, and Figs. 31, 32, and 33 are in angular perspective (§§ 72 and 73).

Construction. The center of the picture is e^v , $G. L.$ is the floor line, and $T Q Z$ is the ceiling line.

(*b*) Fig. 30. This cube is in parallel perspective (§ 72); hence four edges (P) vanish at e^v , and eight edges (A and B) are parallel to the picture plane. A point of distance for P lines is $V^{45^\circ L}$ (or $V^{45^\circ R}$), since $e^v V^{45^\circ L} = e^v e = 3.5'' (= 7' \text{ to scale})$. Draw $I^{P1} e^v$, the indefinite perspective of one of the edges of the base; on $G. L.$, the line of measures corresponding to $V^{45^\circ L}$, lay off $I^{P1} I^{L1} = 0.5'' (= 1' \text{ to scale})$, draw $I^{L1} V^{45^\circ L}$ intersecting $I^{P1} e^v$ at b ; then b is $1'$ behind the wall or picture plane. Lay off $I^{P1} I^{P4} = 2'' (= 4' \text{ to scale})$, draw $I^{P4} e^v$, draw A^1 parallel to $G. L.$ intersecting $I^{P4} e^v$ at a ; then A^1 is the perspective of the near edge in the floor. The diagonal of the base inclined to the left vanishes at $V^{45^\circ L}$, because it is a horizontal line inclined at an angle of 45° to the picture plane; hence draw $b V^{45^\circ L}$ intersecting $I^{P4} e^v$ at k , and the perspective of the edge P^4 is determined. Draw A^4 parallel to A^1 and intersecting P^1 at f , thus completing the perspective of the base. We might have laid off $I^{L1} j = 2''$ and drawn $j V^{45^\circ L}$ intersecting P^1 at f , etc. The line $I^{P1} m$ perpendicular to $G. L.$ is the initial line of the face P^1, B^1 , and hence a line of measures for B^1 . Lay off $I^{P1} I^{P3} = 2'' (= 4' \text{ to scale})$, draw $I^{P3} e^v$ and B^1 , which intersect each other at c ; then $b c$ is the perspective of the edge B^1 . Draw B^2, B^3, B^4 perpendicular to $G. L.$ and parallel to B^1 ; also draw P^2, A^2, P^3 , and A^3 to complete the figure (see § 95).

The diagonals of the upper and lower faces vanish at V^L and V^R (see before). The diagonals Z of the side faces vanish at V^Z , since $e^v V^Z = e^v V^L = 3.5''$, and the other two diagonals (not shown) vanish at V^Y . Since Z^1 and Z^2 vanish at V^Z , and

A^1 and A^2 are parallel to the picture plane and horizontal, the vanishing trace of the plane $A^1 Z^1 Z^2$ is TQZ ; and since L^1 vanishes at V^L , and B^1 and B^4 are vertical, the vanishing trace of the plane $L^1 B^1$ is TLQ ; hence V^Q , the intersection of these two vanishing traces, is the vanishing point of the diagonal Q^1 of the cube (§ 66). Similarly the diagonal W^1 vanishes at V^W , etc.

(c) *Measurement of the Diagonals.* The vanishing trace of a plane containing Z^1 is TYZ ; hence D^Z , found by laying off $V^Z D^Z = V^Z V^L = 4.95''$ (§ 84), is a point of distance for Z^1 . Since $I^{P^1}m$ is the initial line of the plane of the face $P^1 B^1$ (see before), it is the line of measures (§ 87) corresponding to D^Z ; hence draw $D^Z bn$ and $D^Z lm$ intersecting $I^{P^1}m$ at n and m respectively; then $nm = 2.83''$ is the true length of the diagonal Z^1 . The vanishing trace of a plane containing R^1 is the horizon, since R^1 is horizontal; hence the required point of distance is $D^{45^\circ R}$, found by laying off $V^R D^R = V^Z D^Z = 4.95''$. $G.L.$ is the line of measures; hence draw $D^R fp$ and $D^R aq$ intersecting $G.L.$ at p and q respectively; then $pq = nm = 2.83''$ is the length of R^1 required. Since the diagonals W^1 and Q^1 intersect, TWQ is the vanishing trace of a plane containing W^1 , and D^W , found by laying off $V^W D^W =$ the hypotenuse of a triangle of which $V^W e^v$ and ee^v are the sides $= 6.06''$, is the point of distance. Since I^{P^1} is the initial point of P^1 , it is one point of the initial line of the plane W^1, Q^1 , which contains P^1 ; hence $I W^1 C^1$ parallel to TWQ is the line of measures. Draw $D^W fr$ and $D^W hi$ intersecting $I W^1 C^1$ at r and i respectively; then $ri = 3.46''$ is the length of W^1 . TLQ is the vanishing trace of a plane containing Q^1 , and D^Q is a point of distance, since $V^Q D^Q = V^W D^W$. The line $I L^1 Q^1$ through I^{L^1} (the initial point of L^1) is the line of measures; hence draw $D^Q bu$ and $D^Q sw$; then $uw = ri =$ the length of Q^1 . The diagonal C^1

is parallel to the picture plane, and also to TWQ by construction, and is contained in the plane $P^1C^1P^3$. Hence IW^1C^1 is the line of measures, and the initial points of P^1 and P^3 determine the length of C^1 ; hence $IP^1IP^3 = 2.83'' = bh$ in perspective. (See § 97.)

In order to either lay off distances on a line in perspective or to measure the perspective of a line, it is essential to find first the initial point of the line, or the initial line of a plane containing the given line. When by the nature of the drawing the initial point of the line is given, we may at once proceed to lay off the distance on the initial line (line of measures) of any plane which contains the line, using a point of distance on the vanishing trace of this plane. But in the majority of problems the portion of the line already drawn or to be measured does not intersect the picture plane (*i. e.*, initial point undetermined); and as this initial point is unnecessary (except when we wish to measure the distance along the line from the picture plane), we may determine the line of measures by the initial point of any line in the plane which is most readily found. The plane to be chosen is usually the one which contains a number of lines of the figure, for its initial line may be used as a line of measures for all lines in the plane.

Take, for example, the measurement of the diagonal W^1 : From previous construction we have the initial point (IP^1) of P^1 , and we know that P^1 and W^1 intersect at f ; hence we may at once draw the line of measures IW^1C^1 parallel to TWQ , which contains the vanishing points of P^1 and W^1 .

If we use the plane $hkf c$, the construction is more difficult. This plane vanishes at $G.L.$, because V^w is on $G.L.$ and A^1 and A^4 are parallel to the picture plane. A point of distance is D^w (on $G.L.$). From previous construction we do not know the initial point of any line in this plane; hence we may as well

find the initial point of W^1 . R^1 is the perspective of the projection of W^1 on the floor (through $G. L.$), and I^{R^1} is the initial point of R^1 ; hence $I^{R^1} I^{W^1}$ perpendicular to $G. L.$ is the initial line of a plane containing R^1 and W^1 ; hence I^{W^1} is the initial point of W^1 , being the intersection of the line with the initial line of a plane containing the line. Through I^{W^1} draw $I W^1 A^4$ parallel to $G. L.$ for the line of measures, draw $D^W h d$ and from d lay off $dg = 3.46''$, draw $g D^W$ intersecting W^1 at f ; then $dg = 3.46'' = hf$ in perspective. The same result would have been obtained by drawing $I W^1 A^4$ through the initial point of the diagonal $h h$, which is at the intersection of $h h$, $I P^4 Z^2$, and $I W^1 A^4$.

NOTE: The two problems "to measure a line in perspective" and "to lay off distances on the perspective of a line" are *identical* as far as the *drawing* is concerned. In the first we draw lines through the extremities of the perspective and find the intercept on the line of measures; in the second we draw lines from the extremities of the distance measured on the line of measures and find the intercept on the perspective. In regard to the line of reasoning to be employed in each problem one may be called the inverse of the other.

(d) Fig. 31. (See analysis.) As one edge (A^1) is in the picture plane (wall) and one edge is in the floor, and the face containing them is inclined 45° to the floor, A^1 is above $G. L.$ a distance $= 1.41'' = \sqrt{2}$ (to scale); hence lay off $1.41''$ above $G. L.$ and draw A^1 parallel to $G. L.$ and $2''$ in length. The Z edges, being inclined upwards at an angle of 45° to the picture plane, and being in profile planes, vanish at V^Z (see before), and the Y edges, being inclined downwards, vanish at V^Y .

Draw the indefinite perspectives Y^1 , Y^2 , Z^1 , and Z^2 of the four edges intersecting A^1 . Through b draw $I B^1 Z^2$ perpendicular to $G. L.$; $I B^1 Z^2$ is the initial line of the plane contain-

ing Y^2 and Z^2 , and hence a line of measures for these lines. D^2 is the point of distance (see before) corresponding to this line of measures. On $I B^1 Z^2$ lay off $b u = 2''$, draw $u D^2$ intersecting Z^2 at f ; then $b f$ is the perspective of the upper right edge. Since the diagonal of the right face which runs from f is vertical, draw B^2 perpendicular to $G. L.$; it intersects Y^2 at c , hence $b c$ is the perspective of the edge Y^2 . The remainder of the construction follows directly, as it is unnecessary to make other measurements.

(e) *Measurement of the Diagonals.* A line of measures for B^1 is $I B^1 Z^2$ (see before). Since B^1 is parallel to the picture plane, take any point on $T Y Z$, as D^2 , for a point of distance; draw $D^2 f u$ and $D^2 c m$ intersecting $I B^1 Z^2$ at u and m respectively; then $m u = p q$ (Fig. 30) $= 2.83'' =$ the length of the diagonal B^2 . The diagonal C^2 is parallel to the picture plane. The initial point of a perpendicular to the picture plane through f is n , because $I B^1 Z^2$ is the initial line of the profile plane $Z^2 Y^2$; hence $I C^1$ parallel to C^1 is the initial line of a plane containing C^1 , and therefore a line of measures. Draw $e^v a p$ intersecting $I C^1$ at p , then $n p = r i$ (Fig. 30) $= 3.46''$. The diagonal G^1 is horizontal and vanishes at V^G ; hence the vanishing trace of a plane containing G^1 is the horizon, and D^G , found by laying off $V^G D^G = V^G V^Y = \frac{3.5}{\cos \theta} = 4.29''$, is the point of distance. A^1 is the line of measures, hence draw $D^G g q$ intersecting $I G^1 M^1$ (or A^1) at q ; then $b q = p n = 3.46''$ is the length required. The M diagonals of the two faces inclined down from A^1 and A^3 vanish at V^M , found by laying off $V^Y V^M = e^v V^W = 4.95''$, since these diagonals are in the faces of the cube inclined 45° to the picture plane. Similarly the N diagonals vanish at V^N . The diagonal M^1 is in the plane $A^1 A^2$; hence $I G^1 M^1$ (A^1) is the initial line and line of meas-

ures. A point of distance corresponding to this line is on $G. L.$ at a distance from $V^M = 7''$, hence draw $D^M a r$; then $b r = 2.83'' = b a$ in perspective. The diagonal M^2 is in the plane $A^3 A^4$. The initial point of a line (Y^4) in this plane is $I Y^1$, hence $I M^3 Y^4$ parallel to A^3 is a line of measures; then by using D^M we obtain $x y = 2.83''$ for the length of M^2 .

A point of distance for N^1 is D^N (on $G. L.$), found by laying off $V^N D^N = 7''$; then $d t = 2.83''$ is the length of N^1 .

(f) Figs. 32 and 33. The two upper cubes are symmetrically placed with reference to $T Y Z$, and similarly placed with reference to the picture plane and ceiling; hence the explanation for one suffices for the other.

One edge of the cube is vertical and $1.5'$ behind the picture plane. To find this edge draw the perpendicular $k e^v$, lay off $k z = 0.75''$, and draw $z V^{45^\circ R}$ intersecting $k e^v$ at a ; then a is $0.75'$ behind the picture plane, and the vertical line B^1 is the indefinite perspective of the edge. The L and R edges are horizontal and inclined at an angle of 45° to the picture plane; hence they vanish at $V^{45^\circ L}$ and $V^{45^\circ R}$ respectively. Draw L^1 and R^1 . A point of distance for L^1 is $D^{45^\circ L}$, and $T Q Z$ is the line of measures, since the face $L^1 R^1$ is in the ceiling. Draw $D^L a m$, from m lay off $m n = 2''$, draw $n D^L$ intersecting L^1 at d ; then $a d$ is the perspective sought. A line of measures for B^1 is $I U^1 B^1$, hence lay off $z p = 2''$ and draw $p V^R$ intersecting B^1 at b ; then $a b$ in perspective $= 2''$. A line of measures for R^1 is $T Q Z$ and the point of distance is D^R , hence draw $D^R a r$ and from r lay off $r j = 2''$, draw $j D^R$ intersecting R^1 at c ; then $a c$ is the perspective of the edge required. The cube is completed by drawing R^4, L^4 ; then B^2, R^2 , etc.

(g) *Measurement of the Diagonals.* The U diagonals vanish on $T W R$ at V^U (not shown), $4.95''$ below V^R . A point of distance, D^U , is found by laying off $V^U D^U = 7''$. The line

of measures for U_2 is zq , hence draw $D^U aw$ and $D^U fq$ intersecting zq at w and q respectively; then $wq = pq$ (Fig. 30) = $2.83''$ = the length of the diagonal U_2 . The diagonal J^1 vanishes at V^J (not shown, $4.95''$ above V^R), a point of distance is D^J on TWR , and the line of measures is zq ($I U^1 B^1$).

It is evident that $D^J c$ intersects zq beyond the limits of the drawing, hence we employ a point of half distance $D^{1/2J}$, found by laying off $D^J D^{1/2J} = 1/2 D^J V^J = 3.5''$. Draw $D^{1/2J} cs$ and $D^{1/2J} bx$ (the point x is just above L^1) intersecting zq at s and x respectively; then $sx = \frac{wq}{2} = 1.42''$ = one half the length of J^1 . The diagonal O^1 vanishes at V^O , and a point of distance is found by laying off $V^O D^O = V^O V^{45^\circ}$. Since O_2 is in a profile plane which contains the edges B^1 and B^2 , a perpendicular to the picture plane through a determines the line of measures $I O^1 B^1$ parallel to TYZ . Draw $D^O ay$ and $D^O lh$ intersecting $I O^1 B^1$ at y and h respectively; then $yh = ri$ (Fig. 30) = $3.46''$. The length of the diagonal K_2 is gi (on TWR) = $2.83''$.

(See § 105 for construction of the circles inscribed on the faces of the cube in Fig. 33.)

99. Successive Perspective Plans. It is often necessary in order to save confusion in the drawing and render the construction more simple to make a *perspective plan* of the object at some distance either below or above the desired position.

The plan of a large building, for example, is frequently quite complicated, and as at least half is invisible from a given position, it is unnecessary to show that portion in the finished drawing. The complete plan, however, is needed in order to obtain the details of the roof, the chimneys, etc.

Plate VI, Fig. 29. Angular perspective. Given the vanishing points, etc., as shown, also given a plan. The perspective

drawing is made to the same scale as the plan in this particular case. The plane of the plan is horizontal and the point a is in the picture plane. The points a, b, c , etc., of the plan correspond to the points a^1, b^1, c^1 , etc., of the perspective on the horizontal plane through $G. L.$, and the method of measurement is readily traced.

Take any new ground line, as $G_1. L_1$, parallel to $G. L.$; from a^2 erect a perpendicular to $G. L.$ (and H) intersecting $G_1. L_1$ at a^2 . From a^2 lay out the perspective of the plan by measurements as before. Since a^2 was determined by a perpendicular to $G. L.$ through a^2 , all the other points, as b^2, c^2, d^2 , etc., might have been similarly determined; *e. g.*, draw R^2 , erect the perpendicular $b^2 b^2$ intersecting R^2 at b^2 ; draw L^2 and a perpendicular through c^1 intersecting L^2 at c^2 , etc. Hence it is evident that *measurements* were unnecessary to determine the second perspective plan. The geometric statement of the preceding is: If a cylinder (general sense) be perpendicular to H , all horizontal sections are equal, and the horizontal projections of these sections coincide with the base of the cylinder on H .

100. Problem III. *To find the perspective of a cube. Oblique perspective. Plate VIII, Fig. 38.*

Analysis and Construction. All the faces and edges are oblique to the picture plane (§ 74). Each edge is 2'' long. Let F, R, Q be three edges of the cube intersecting at the point a which is in the picture plane. Assume the vanishing points V^F and V^Q of the edges F^1 and Q^1 respectively; then $T F Q$ is the vanishing trace of the plane of those edges. Assume V^N , the center of $V^F V^Q$, to be the vanishing point of the diagonal N^1 which bisects the angle $F^1 Q^1$. From these assumptions it follows that the center of the picture is on $V^N V^R$ drawn through V^N perpendicular to $T F Q$. Assume e^v as the center of the picture; hence the *axis* (§ 57) is one side of a right-

angled triangle of which the other side and hypotenuse are $e^v V^N$ and $V^N V^F$ respectively. With V^N as a center and radius $V^N V^F$ describe an arc of a circle $V^F e_2 V^Q$, draw $e^v e_1$ perpendicular to $V^N V^R$ intersecting the arc at e_1 , then $e^v e_1$ is the length of the axis. The edge R^1 is perpendicular to the face $F^1 Q^1$ and to the diagonal N^1 ; hence R^1 vanishes on $V^N V^R$. Describe an arc of a circle through $V^N e_1$ with center in $V^N V^R$; then V^R is the vanishing point of R^1 , since $V^N e_1 V^R$ is a right angle, being inscribed in a semicircle.

$e_1 V^R$ is the distance of the station point from V^R ; hence with V^R as a center and $V^R e_1$ as radius describe a circle which is the locus of the points of distance, D^R , for the system R . With V^Q as a center and radius $V^Q e_2$ describe a circle, thus obtaining the locus of D^Q ; and with V^F as a center and radius $V^F e_2$ describe a circle, thus determining the locus of D^F . We may now proceed to draw the cube.

Through a , the vertex in the picture plane, draw Q^1, R^1, F^1 , the indefinite perspectives of the edges which intersect at a . Draw the line of measures $I Q^1 R^1$ through a parallel to $T Q R$, lay off $an = 2''$ and draw $n D^R$ intersecting R^1 at b ; also lay off $at = 2''$ and draw $t D^Q$ intersecting Q^1 at d ; the perspectives of two edges are now determined. Through a draw the line of measures $I F^1 R^1$ parallel to $T F R$, lay off $am = 2''$ and draw $m D^F$ intersecting F^1 at f , thus determining the perspective of the third edge. The drawing is finished as in the preceding problems, as all the necessary measurements have been made.

Instead of assuming e^v we might have assumed V^R ; then e^v may be found as follows: Describe semicircles through V^F, V^N, V^R ; V^Q, V^N, V^R and V^Q, e_2, V^F .

From the points of intersection o and p of these semicircles draw $o V^F$ and $p V^Q$ intersecting at e^v , the center of picture required. Let the student prove this statement.

100a. Second Solution. Fig. 38a. The essential points in either solution are the determination of the position of the station point (having assumed the vanishing points of two edges which are not parallel), and the position of the third vanishing point.

Suppose the vanishing point of the diagonal N^1 does not bisect $V^F V^Q$ (Fig. 38), but has the position shown in Fig. 38a.

To find the position of the station point: On $V^F V^Q$ describe a semicircle which must contain the revolved position (e_1) of the station point, since F^1 and Q^1 are perpendicular. A line from e_1 to V^N must bisect the right angle between F^1 and Q^1 ; hence the angle $V^Q e_1 V^N =$ the angle $V^F e_1 V^N = 45^\circ$. Hence in the triangles ze_1r and re_1x we have the following proportion: $zr:rx = \sin \beta : \sin \alpha = \cos \alpha : \sin \alpha = \cot \alpha$. Through r draw $rc = rx$ and perpendicular to zx , draw zc ; then $\cot \alpha = \frac{zr}{rc}$. The line zc intersects the semicircle at e_1 , which is there-

fore the revolved position of the station point in the picture plane; hence the center of the picture may be anywhere on the line e_1d drawn perpendicular to $V^Q V^F$. The position of the third vanishing point readily follows.

CHAPTER VII.

PERSPECTIVE OF CIRCLES.

101. The problem, to find the perspective of a circle, is the Descriptive Geometry problem, to find the intersection of a cone with a plane (§ 51). In perspective the intersecting plane is the picture plane, and the cone is given by its *vertex* (the station point) and *circular base* in a plane which is usually oblique to the picture plane.

102. *In general the perspective of the center of a circle is not the center of the perspective of the circle*, for the same reason that in general the axis of a cone does not pass through the center of an oblique section.

103. Proposition IV. *If any variable tangent to a central conic section meet two fixed parallel tangents, it will intercept portions on them, whose rectangle is constant and equal to the square of the semi-diameter parallel to them.** Plate VIII, Fig. 39.

Construction. Let ac be a diameter of the ellipse $abcd$, M and Q tangents at the extremities of the diameter ac , and N any other tangent intersecting M and Q at f and g respectively; then $af \times cg = (ob)^2$.

Hence, if we have the tangents to any diameter of an ellipse and any third tangent, the length of the semi-conjugate diameter may be readily found (Fig. 39a). $af = af$ (Fig. 39), $fc = gc$ (Fig. 39). On ac as a diameter describe a circle, draw

* Salmon's Conic Sections.

fm perpendicular to ac , then $(fm)^2 = (ob)^2$ (Fig. 39) $= af \times fc$.

104. If we have six or more tangents to a small ellipse and the corresponding points of tangency, the ellipse may be drawn accurately without finding diameters.

105. Problem IV. *Given a cube in perspective to describe circles on the visible faces.* Plate VII, Figs. 33 and 33a. Method of tangents (§ 104).

Analysis and Construction. For the construction of the cube see § 98f. It is required to inscribe circles (in perspective) in $R^1 R^2$, $R^2 R^3$, and $B^1 B^3$, the visible faces of the cube.

Fig. 33a. To find the tangents and points of tangency (§ 104) inscribe a circle in a square ($2''$ sides) of the same size as that of the face of the cube. Draw the diagonals na and mh and the four tangents to the circle which are perpendicular to these diagonals, and we have eight tangents and eight points of tangency, y, c, d, i, o, k, g , and f , which are to be transferred to the perspective.

Fig. 33. The lengths of L^1 and R^1 were laid off by means of the line of measures TQZ ; mn is the length of L^1 , and rg is the length of R^1 . Hence lay off $np = np$ (Fig. 33a), $po = po$, $oq = oq$ and $qm = qm$. From the points p, o, q draw lines to D^L intersecting L^1 , and thus transfer the distances laid off on mn to L^1 . Similarly for R^1 .

The same subdivision may be effected by proportional measures (§ 90) as follows: Through a draw at , a line of proportional measures, parallel to the horizon, on at lay off ax , xt , etc., $= ax$, xt , etc. (Fig. 33a) respectively, draw tc P^R ; then P^R is a point of proportion distance. From the points of subdivision on at draw lines to P^R intersecting R^1 at j, y , and s , thus determining the same points as before.

To divide B^1 , which is parallel to the picture plane (§ 94),

proceed as follows: Through a (Fig. 33a) draw $ab = a'b'$ (Fig. 33), join $b'h$, from the points of division draw parallels to $b'h$ intersecting ab as shown. Transfer the distances thus obtained directly to B^1 (Fig. 33), and the required subdivision is effected.

Complete the octagon on each face and draw the diagonals to determine the points of tangency not on the edges of the cube. Note the fact that the tangents (Fig. 33a) pk and jc are parallel to mh , and hence have the same vanishing point, and that qi and sf are parallel to na , and hence have the same vanishing point. On each face draw a smooth curve (ellipse) tangent to the octagon at the determined points of tangency, and thus obtain the required solution.

106. Problem V. *To find the perspective of a circle.* Plate VIII, Figs. 40–43a inclusive.

General Analysis. The same horizon, vanishing points, etc., are used for all the figures. The center of the picture is e^v , and the station point is $4''$ in front of the picture plane i. e., $e^v V^{45^\circ} = 4''$. The diameter of each circle is $1.5''$.

(a) *First Case.* When the circle is in a vertical plane, and the perspective of the center is on the principal horizon. Fig. 40.

Analysis. Circumscribe a square about the circle with its sides vertical and horizontal. Assume that the horizontal sides vanish at $V^{45^\circ R}$, thus placing the circle in a vertical plane inclined 45° to the picture plane, and assume that one of the vertical sides is in the picture plane. The circumscribed square will give two parallel tangents to the perspective of the circle, and two tangents which are not parallel; hence diameters of the elliptical perspective are readily found by § 103.

Construction. Let $ab = 1.5''$ be the side of the square in the picture plane, and R^1 and R^2 the indefinite perspectives of the horizontal sides. Since the perspective of the center is to

be on the horizon, $ap = bp$. Through b draw a line of measures, bn , parallel to the horizon, lay off $bm = mn = 0.75''$, and draw mD^R and nD^R intersecting R^1 at f and c respectively. Draw fk and cd perpendicular to the horizon; then $abcd$ is the perspective of the circumscribed square. Since pq is perpendicular to the tangents ab and cd , it is an axis of the ellipse. Through o , the center of pq , draw gor parallel to ab , and lay off $or = og = \sqrt{dq \times ap}$ for the major axis. An ellipse constructed on pq and gr as axes is the perspective of the circle required. e^v in this case coincides with the perspective of the center of the circle; hence the axis of the picture (§ 57) is the axis of the projecting cone of the circle.

(a) *Second Case.* Fig. 40a. *Analysis.* The position of the circle is so chosen that the picture plane cuts a sub-contrary section from the projecting cone. One point of the circle is in the picture plane. A circumscribed square is used as before.

Construction. Let the plane of the circle be inclined at an angle of 60° to the picture plane towards the right. The perspective of the center is $2.31''$ ($4 \tan 30^\circ$) to the left of e^v at D^N , since the axis of the projecting cone must be inclined at an angle of 60° (in this case) to the picture plane in order that the picture plane cut a sub-contrary section. The triangle lying in a horizontal plane through the horizon and formed by the axis of the projecting cone, the horizon, and a diameter of the given circle, is equilateral (in this case); hence to find the point of the circle in the picture plane lay off $D^N r = 0.75''$, and r is the point desired. Through r draw $arb = 1.5''$ for the side of the circumscribed square which is in the picture plane. Draw N^1 and N^2 . Through b draw bm parallel to the horizon, draw mD^N intersecting N at c , and draw cd parallel to ab to complete the square. Bisect rf at o ; with o as a center and radius or describe a circle, which is the perspective required.

(a) *Third Case.* When the circle is in a vertical plane and the perspective of the center is not on the horizon. Fig. 41.

Analysis and Construction. Let the perspective be a duplicate of Fig. 40, except that the center is below the horizon. Let $a b$, on $a b$ (Fig. 40) produced, be the side of the circumscribed square in the picture plane, draw R^1 and R^2 limited by $d c$ (Fig. 40) produced. Then $a b c d$ is the perspective of the circumscribed square. Through p , the center of $a b$, draw R^3 . Produce $a b$ to t , making $b t = c q$, on $p t$ describe a semi-circle, draw $b r$ perpendicular to $p t$; then $b r$ is the length of the semi-diameter conjugate to $p q$ (§ 103). Through o , the center of $p q$, draw $g o l$ parallel to $a b$, and lay off $o l = o g = b r$ for the diameter conjugate to $p q$. An ellipse constructed on these diameters is the perspective of the given circle.

(b) *First Case.* When the circle is in a horizontal plane. Fig. 42. *Analysis.* In order to obtain diameters of the perspective easily the sides of the circumscribed square should be parallel to, and perpendicular to, the picture plane.

Construction. The perspective of the circumscribed square is $a b c d$, found by employing a point of one-half distance, $D^{1/2P}$, instead of V^R , for the measurement of the sides perpendicular to the picture plane. The parallel tangents (see before) are $b c$ and $a d$, and $c d$ is a third tangent. A mean proportional between $c f$ and $d j$ is $c g$; hence through i , the center of $j f$, draw $t i r$ parallel to $a d$, and lay off $i r = i t = c g$. On $f j$ and $t r$ as conjugate diameters construct an ellipse which is the required perspective.

(b) *Second Case.* When the circle is in a horizontal plane. Figs. 43 and 43a.

Fig. 43 is a duplicate of Fig. 43a in perspective. The circle is circumscribed by an octagon (see § 105), and an ellipse

is drawn through the points of tangency in perspective. As the construction is like that of § 105, no description is given.

107. Apparent Distortions. Fig. 44 shows the perspectives of four horizontal circles constructed as in § 105. The circles are equal and their centers are on the same straight line. The perspectives present a twisted or distorted appearance, although the drawing is accurate, due to the fact that in looking at the drawing the eye is not usually placed exactly at the station point (§ 55). This distortion is much more apparent in curves than in straight lines, although it exists equally in both. From an *artistic* standpoint this figure shows the limitations of One-plane Perspective. In order to remedy this apparent distortion and make the drawing "look right" from other positions than the station point, the perspectives should be turned so that their axes are parallel. The amount of change necessary to effect the correction depends greatly upon the artistic sense of the draftsman, and no rules will be given for guidance, except that the drawing should first be made accurately and the changes made afterwards.

When the perspectives of the centers of circles are on the horizon (see Figs. 40 and 40a), no correction is needed, for the axes of the ellipses are parallel.

108. Problem VI. *To find the perspective of a sphere.* Plate IX, Fig. 45.

Analysis. This problem in Descriptive Geometry would be stated thus: To find the intersection with V of a cone tangent to a given sphere, the vertex of the cone being a given point. To solve the problem in perspective we employ partly the direct method of Descriptive Geometry and partly the method of measurements heretofore given.

Consider the picture plane to be the V coördinate plane, and the H coördinate plane to be a horizontal plane through G . L .

The sphere, 1.5'' diameter, is in the second dihedral angle, and the projections coincide. The vanishing points, etc., are as shown, and the station point is 3'' in front of the picture plane.

Construction. Pass a cone tangent to the sphere having its vertex at e . B^H, D^H are the H projections of the contour elements, and the broken line to the right of, and parallel to, $f^H c^H g^H$ is the H projection of a chord of the circle of tangency which is parallel to H in a horizontal meridian plane of the sphere. Project the sphere and tangent cone on a new V plane through G_1, L_1 , parallel to the axis of the cone; then $b^{V_1} c^{V_1} d^{V_1}$ is the new V projection of the apparent contour of the cone, and $b^{V_1} c^{V_1} d^{V_1}$ is the projection and diameter of the circle of tangency. The H projection of this diameter is $b^H c^H d^H$, which is an axis of the H projection of the circle. Draw $f^H c^H g^H$ perpendicular to $d^H b^H$, and lay off $c^H f^H = c^H g^H = c^{V_1} b^{V_1}$; the ellipse constructed on these lines for axes is the H projection of circle of tangency. The problem now becomes: To find the perspective of a circle which is in an oblique plane. The plane Q perpendicular to the axis of the tangent cone is a plane parallel to the plane of the circle of tangency. $T N Q$ parallel to $V Q$ is the vanishing trace of the system Q , and hence the vanishing trace of the plane of the circle of tangency.

Consider a square circumscribed about this circle with two of its sides horizontal; then the other two sides are lines of greatest declivity of the plane of the circle. The horizontal sides vanish at V^N , and the inclined sides at V^Q (not shown), the intersection of $T N Q$ and $T O Q$; since the H projections of the inclined sides being perpendicular to $H Q$, vanish at V^O . D^N , found by laying off $V^N D^N = p e^H$, is a point of distance for N lines, and D^Q (not shown), found by laying off $e^V D^Q = V^Q e$, is a point of distance for Q lines.

Fig. 45a shows an octagon circumscribed about a circle which is equal to the circle of tangency and gives the measurements necessary to draw the perspective (§ 105). The perspective of the H projection of the center o of the sphere is o^{1st} , the intersection of $(e^{1st} o^{1st}, e^{1st} n)$ with the picture plane (V). Horizontal lines perpendicular to the horizontal sides of the circumscribed square, *i. e.*, lines parallel to the H projections of the inclined sides, vanish at V^0 (see before), and their point of distance is D^0 .

Through o^{1st} draw $D^0 o^{1st} j$, lay off $j l = h i$, $j q = i k$, and $j n = h k = k y$, draw $l D^0$, $q D^0$, and $n D^0$ intersecting O^1 , the perspective of the H projection of a line through the center of the circumscribed square parallel to the inclined side, at d^{1st} , c^{1st} , and b^{1st} respectively. These points are the perspectives of the H projections of d , c , and b respectively. To find the perspectives of the points b , c , d , take any point on the horizon, as D^K , for a point of distance, and draw $D^K b^{1st} v$, $D^K c^{1st} w$, and $D^K d^{1st} x$ intersecting $G. L.$ at v , w , and x respectively. From these points erect perpendiculars to $G. L.$, lay off distances equal to $b^{1st} y$, $c^{1st} k$, and $d^{1st} h$, and from the points thus obtained draw lines to D^K intersecting O^1 at b^1 , c^1 , and d^1 respectively, which are the perspectives of b , c , and d respectively of the circle of tangency, or the points b , c , and d of Fig 45a. Draw N^1 , N^2 , and N^3 , the indefinite perspectives of the horizontal sides of the circumscribed square, and of a line parallel to them through the center of the circle. Since N^2 is horizontal, its line of measures is $s z$ parallel to $G. L.$, and at a distance above $G. L.$ equal to $h d^{1st}$. Draw $D^N d^1 u$, lay off $u s = u z = d f$ (Fig. 45a), and draw $s D^N$ and $z D^N$ intersecting N^2 at m and r ; then $m d^1 r$ is one side of the circumscribed square in perspective. Draw Q^1 and Q^2 to complete the perspective of the square. The perspective of the octagon (not shown) is now

readily found. The ellipse as shown is the perspective of the sphere.

Alternate Method. After finding the two projections (only H projection shown) of the circle of tangency of the projecting cone we may use the direct method of § 77 to find the perspective of the circle; *i. e.*, find the intersection (base) of the cone with the picture plane (V). Let a be any point on the circle of tangency, through e (the station point) and a draw the line F , which intersects the picture plane at a^1 ; hence a^1 is the perspective of a . (See § 107.)

109. Method of Coördinates. If the position of a point relative to three rectangular axes be known, its perspective may be readily determined. This method for the determination of perspectives is known as the *Method of Coördinates*. It is used to find the perspective of any irregular object which cannot be readily determined by other methods, and for sketching in contours.

Construction. Plate IX, Fig. 46. Let X, Y, Z be the rectangular axes corresponding to the axes of Analytic Geometry. (See § 22.)

Two of the axes are in the picture plane, and the third passes through their intersection perpendicular to the picture plane. The axes are divided according to any convenient scale, and parallels to the axes are drawn through the points of division.

To determine the perspective of a point the coördinates of which are $+3, -3, +2$ proceed thus: Plus 3 means that the point is in a vertical plane through $3p$, -3 that the point is in a vertical plane through $3q$, and $+2$ that the point is in a horizontal plane through $2r$; hence the perspective of the point is a , the intersection of the three planes.

CHAPTER VIII.

PERSPECTIVE OF SHADOWS.

110. For definitions, etc., concerning shadows see §§ 5-17 inclusive.

111. Vanishing Point of Rays. The *vanishing point of rays* of light is in the vanishing trace of vertical planes inclined to the right at an angle of 45° to the picture plane, and at a distance below the horizon equal to the length of the axis of the picture (see Fig. 1, page 3), since rays are equally inclined to H and V (or picture plane) and their projections make angles of 45° with $G. L.$

The H projections of rays vanish at $V^{45^\circ R}$.

112. Determination of the Perspectives of Shadows. The determination of the perspective of the shadow of any right-line object involves usually the determination of the vanishing point of the line of intersection (§§ 11 and 13) of a plane of rays passed through each division of the line of shade with the plane which receives the shadow; *e. g.*, if the shadow is on H and the line of shade is vertical, the vanishing point of the shadow is $V^{45^\circ R}$, because this point is the intersection of the vanishing trace of H with the vanishing trace of a plane of rays through the vertical line of shade (§ 66).

113. Perspective of the Shadow of a Point. The following is a general method for finding the perspective of the shadow of a point on a plane: Through the perspective of the point draw the perspective of a ray, and through the perspective of the projection of the point on the plane draw the per-

spective of the projection of a ray on the plane; the intersection of these two lines is the perspective of the shadow of the point on the plane.* This method follows directly from the fact that the intersection of a line with a plane is the intersection of the line with its projection on the plane.

114. Problem VII. *To find the perspective and the perspective of the shadow of a vertical prism mounted on a parallelopiped.* Angular perspective. Plate X, Fig. 47.

Analysis. The parallelopiped measures $2'' \times 2'' \times 1.06''$, and prism $0.5'' \times 0.5'' \times 1.39''$. The horizon is the lower of the two horizons shown in the figure. The vertical faces of the two bodies are inclined 30° and 60° to the picture plane, and the vanishing points of the principal lines are as shown, except V^M , which is not within the limits of the drawing. The station point is $4''$ in front of the picture plane. As the method for finding the perspective is similar to that given in previous problems, no explanation is necessary, and we may immediately proceed to find the perspective of the shadow.

The *line of shade* is determined by inspection, as in Shades and Shadows. Part of the shadow is on H and part is on the parallelopiped.

Construction. The line of shade of the parallelopiped is composed of the edges A^1 , U^2 , M^3 , A^3 , and the two left edges in H . Rays vanish at V^X , and the H projection of rays vanish at $V^{45^\circ R}$.

The vanishing trace of the plane containing A^1 and a ray is $T R X$, and the vanishing trace of the plane on which the shadow of A^1 is cast is the horizon, since the shadow of A^1 is on H ; hence V^R , the intersection of $T R X$ and the horizon, is the vanishing point of the shadow. Through a draw R^1 , the

* Descriptive Geometry, A. E. Church.

indefinite perspective of the shadow of A^1 on H , and through f draw the perspective of a ray intersecting R^1 at f_s ; then af_s is the perspective of the shadow required. U^2 vanishes at V^U ; hence the vanishing trace of a plane of rays through U^2 is $T U X$. The shadow of U^2 is on H and vanishes at V^U , since U^2 is horizontal; hence U_s^2 is the indefinite perspective of the shadow. The perspective of a ray through o intersects U_s^2 at o_s ; hence $f_s o_s$ is the perspective of the shadow of U^2 on H . Since M^3 is horizontal and vanishes at V^M (not shown), the perspective of the shadow of M^3 on H vanishes at V^M ; hence draw the indefinite perspective $o_s V^M$. It is now evident that the perspective of the shadow of the rear edge, A^3 , is invisible, and therefore it is not drawn.

The line of shade of the prism is composed of the edges A^4 , A_s^4 , $j k$, $k l$, $d c$, and $c b$. Since the upper base, $U^2 M^2$, of the parallelopiped is horizontal, the shadow of A^4 on this plane vanishes at V^R (see before); hence through b draw the indefinite perspective, R^2 , of the shadow. R^2 intersects M^4 at u , the perspective of a ray through u intersects $o_s V^M$ at u_s ; then R^4 limited by the perspective of a ray through j is the perspective of the shadow of A^4 on H .

Or, through b^R draw $b^R V^R$ (R^4) intersecting $o_s V^M$ at u_s , etc. The shadow of $j k$ on H vanishes at V^U , hence draw $j_s V^U$ and $k V^X$ intersecting at k_s , then $j_s k_s$ is the perspective of the shadow of $j k$.

The line $k l$ vanishes at V^M , hence draw $k_s V^M$ and $l V^X$ intersecting at l_s , then $k_s l_s$ is the perspective of the shadow of $k l$. Since A^4 is vertical, draw R^4 ($d^R l_s V^R$) to complete the shadow on H in perspective. The perspective of the shadow of A^4 on the upper base of the parallelopiped is R^4 , found by drawing $d V^R$.

The perspective of the visible shadow on H is R^1 , U_s^2 , $o_s u_s$,

$R^s, j_s k_s, k_s l_s, R^4$, and M^1 , which is its own shadow. The perspective of the visible shadow on the upper base of the parallelopiped is bu and a small part of R^s .

115. Problem VIII. *Given a vertical rectangular column near a small building having a projecting roof: To find the perspective and the perspective of the shadow.* Angular perspective. Plate X, Fig. 48.

Analysis. The vanishing points, etc., are as shown, and the station point is $4''$ in front of the picture plane. One edge of the column is in the picture plane, and from the center of the rear edge of the upper base a guy-line (F) extends to the ground. The column is in $30^\circ, 60^\circ$ perspective, and the building is in 45° perspective as shown. The special problem is to determine the perspectives of the shadows of the column and guy-line on the building.

Construction. Let $abcd, f, B^1, B^2$ be the perspective of the column with the guy F^1 running from f to the ground, which is a horizontal plane through $G. L.$, at g . Also, let the perspective of the building be as shown, with $m^H j^H l^H k^H m^H$ for the perspective plan (§ 99) of the roof. The vanishing trace of the plane of the left side of the roof is $T L J$, and of the right side is $T L W$. V^J is on $T W R$ above the horizon.

The shadow of the building. The line of shade is composed of the lines $J^1, J^2, L^3, L^2, A^2, A^3$, and $q v^H z l^H$. The indefinite perspective of the shadow of A^2 is $v^H V^H$. The perspective of the shadow of m (the corner of the roof) is m_s , found by § 113, *i. e.*, the intersection of the perspective of a ray through m with the perspective of the H projection of a ray through m^H . Since L^1 is horizontal and vanishes at V^L , its shadow on H vanishes at V^L , hence through m_s draw $m_s V^L$ intersecting $v^H V^H$ at p_s , then $v^H p_s$ is the perspective of the shadow of A^2 . The vanishing trace of a plane of rays through J^2 is $T W R$, hence

the shadow of J^2 on H vanishes at V^R . Draw R^3 , through u draw the perspective of a ray intersecting R^3 at u_s , then $m_s u_s$ is the perspective of the shadow of J^2 . L^2 vanishes at V^L , hence $n_s V^L$ is the indefinite perspective of its shadow. The perspective of a ray through o intersects $n_s V^L$ (L_s^2) at o_s . Through o_s draw R^1 for the perspective of the shadow of J^1 . The remainder of the shadow on H is invisible. The vanishing point of the shadow of L^1 on the right side of the building is V^X , the intersection of $T W R$ and $T L X$, hence through p , the intersection of the plane $A^1 A^2$ with L^1 , draw X^1 for the perspective of the required shadow on the right side of the building. Through v draw $v V^L$ for the perspective of the shadow of L^1 on the left side.

Shadow of the Column. Since B^1 and B^2 are vertical their shadows on H vanish at V^R , hence through a and b draw R^1 and R^2 respectively. The perspectives of rays through x and c intersect R^1 and R^2 at x_s and c_s respectively, thus determining the perspectives of the shadows of B^1 and B^2 on H . Since $x d$ vanishes at V^Q , its shadow on H vanishes at V^Q , hence draw $x_s V^Q$; and since $d c$ vanishes at V^Q , its shadow on H vanishes at V^Q , hence draw $V^Q c_s d_s$ intersecting $x_s V^Q$ at d_s . We now have the complete shadow of the column on H in perspective. The intersections of planes of rays through B^1 and B^2 with the side $A^2 A^3$ of the building are parallel to B^1 and B^2 and hence are parallel in perspective, therefore draw B_s^1 and B_s^2 for the perspectives of the shadows of B^1 and B^2 on the side of the building. Since $m^n k^n$ is the perspective plan of L^1 , the plane $m^n m k^n$ is parallel to $A^2 A^3$, hence erecting perpendiculars from s^n and r^n , the shadows s and r of B^1 and B^2 respectively on L^1 are determined. The vanishing trace of a plane of rays through B^1 or B^2 is $T W R$ and the vanishing trace of the left side of the roof is $T L J$, hence the vanishing point of the

shadows of B^1 and B^2 on this side of the roof is V^J (not shown), the intersection of TWR and TLJ . Through s and r draw J^3 and J^4 respectively for the required perspectives. Similarly the perspectives of the shadows of B^1 and B^2 on the right side of the roof vanish at V^W , the intersection of TWR with TLW , hence through i and h draw iV^W and hV^W respectively for the required perspectives. These two lines are limited by jL .

Shadow of the Guy. F vanishes at V^F , hence the vanishing trace of a plane of rays through F is TFN . The vanishing point of the shadow of F on H is V^N (not shown), the intersection of TFN and the horizon, hence draw N^1 for the perspective of the shadow on H . Or, find the perspective of the shadow of f on H and join with g . The shadow of F on the H projecting plane of L^1 , or the plane $m^m m k k^m$, vanishes at V^x , hence tu is the perspective of the shadow of F on that plane. The vanishing point of the shadow of F on the left side of the roof is the intersection of TFN and TLJ (not shown, beyond the left border), hence through u draw E^1 . The vanishing point of the shadow of F on the right side of the roof is V^U , the intersection of TFN and TLW , hence from the point where E^1 intersects L^2 draw a line to V^U and limit it by the intersection with jL . Or, to find the shadow of the guy on the roof, produce W^2 to y and draw ywV^L , which is the perspective of the intersection of the plane of the right side of the roof with H . N^1 intersects yV^L at w , hence through w draw V^Uw intersecting L^2 , etc.

116. Illustration. Plate XI shows the perspective of a C. B. & Q. R. R. standard furniture car drawn by the methods outlined in the preceding paragraphs. The scale is $\frac{3}{8}'' = 1'$. The principal lines are in 30° , 60° angular perspective; the 30° vanishing point is about $23''$ to the right of e^V , and the 60° vanishing

point is about 7" to the left of e^v . e^v is 2.5" above O (see drawing), and the station point is 13" (nearly) in front of the picture plane. No part of the car or track is in the picture plane.

Problems.

Each problem may be stated in six different ways, depending upon the position of the object relative to the picture plane. (a) Parallel perspective, (b) Angular perspective, (c) Oblique perspective; (1) Part of the object is in the picture plane, (2) No part of the object is in the picture plane. When the statement of the problem is succeeded by the letters (a 1), etc., it means that the drawing is to be made in parallel perspective (a), and that a point or line of the object is in the picture plane (1), etc. Each problem is to be solved in the positions (a 1), (a 2), (b 1), and (b 2), unless otherwise stated.

All dimensions are to be given and the perspectives are to be drawn to scale.

1. Find the perspective of a pyramid, base on H .
2. Find the perspective of an inverted pyramid, base parallel to H .
3. Find the perspective of a vertical hexagonal prism.
4. Find the perspective of a hexagonal prism resting on one face in H .
5. Find the perspective of a vertical cross.
6. Find the perspective of a square column surmounted by a square abacus.
7. Find the perspective of a rectangular box with its lid partly raised. (c 1), (c 2), etc.
8. Find the perspective of a bookcase containing three shelves.
9. Find the perspective of a square-topped table with four square legs.

10. Find the perspective of a flight of steps.
11. Find the perspective of a triangle situated in an oblique plane. (*c* 1), (*c* 2), etc.
12. Find the perspective of a vertical circular cylinder.
13. Find the perspective of a circular cone, axis vertical.
14. Find the perspective of an ellipsoid of revolution, axis vertical.
15. Find the perspective of any building.
16. Find the perspective of a groined arch. (*a* 1), (*a* 2).
17. Find the perspective and the perspective of the shadow in any one of the preceding problems.

APPENDIX.

Shading.

117. Hypothesis.* (*a*) All objects to be shaded are to be considered *unpolished*; the character of the surface being similar to that of a plaster cast.

(*b*) The sun is the only source of direct light and is fixed in position (§ 5).

(*c*) Objects are also illuminated by *reflected light* from the minute particles which are everywhere present in the atmosphere.

Were it not for reflected light the shade of an object would be black; hence these particles illuminate the shade or portions in shadow and cause them to be visible. Reflected light, being much less intense than direct light (*b*), does not cause shadows and does not affect the line of shade.

(*d*) For the illumination of the *shade* the minute reflecting particles (*c*) may be considered, with but slight error, to be concentrated in a point diametrically opposite the sun or source of light. (See *f* and *g*.)

(*e*) The only kinds of light which are supposed to reach an object are from the source (*b*) and from atmospheric particles (*c*). All surrounding objects which might reflect light and thus

* This method of shading is due M. Jules Pillet, École Nationale des Ponts et Chaussées, to whose works the student is referred for a more extended treatment embracing the effect of distance, color, etc.

alter the appearance of the given object are considered to be removed.

(f) *The apparent illumination of an object depends only on the intensity of the source of light and the angle which the rays make with a normal to the surface.*

(g) The illumination of any element (any very small area) of a surface is proportional to the cosine of the angle which the incident ray makes with a normal to the surface at that element.

118. Deductions. Using the sphere as a typical surface, we make the following deductions from § 117. (See Plate XII, Fig. 49.)

(a) The apparent darkest part of the sphere is the line of shade 0, because the cosine of the angle between an incident ray and normal is a minimum, *i. e.*, = 0, at any point of the line; the apparent lightest part is the point 8, because the cosine of the angle between the incident ray and a normal at this point is a maximum, *i. e.*, = 1. From 8 to 0 the shading gradually increases in depth, and from 0 to -5 it gradually decreases in depth; the part 0 to -5, however, is much darker than the part 0 to 5, since it is unilluminated by direct light.

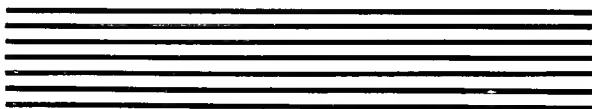
(b) If any object be placed so as to exclude light from the sphere, the shadow is as much darker as that portion would be lighter without the interposed object, for the shadow is illuminated only by reflected light from atmospheric particles.

119. Method of Shading. Plate XII, Figs. 49 and 49a. Let the diameter of the sphere which coincides with a ray be divided into sixteen equal parts, and let planes be passed perpendicular to the diameter (ray) through the points of division. Fig. 49a shows the lengths of the axes of the projections of the circles cut by the secant planes.

Call the line of shade 0, the ellipses (in projection) which

are illuminated by direct light 1, 2, 3, 4, etc., and the ellipses (in projection) which are illuminated only by reflected light — 1, — 2, — 3, — 4, etc. We are now ready to apply the tints of india ink to produce the proper shading.

(a) To obtain the tint to be applied to the shade rule a small sheet of white paper with equidistant lines, thus :



so that the width of each line is one half the width of the white space between any two lines. In a small beaker place six portions of water (a brushful may serve as a measure) and enough india ink to obtain a shade equivalent in depth to that produced by the equidistant lines on the ruled sheet. Call this tint No. 0 and apply to the shade of the object, *i. e.*, between the 0 line and the right contour. Add *one* portion of water to tint No. 0, call this tint No. 1, and apply between the lines 1 and — 1 after the 0 tint is dry. Add *two* portions of water to No. 1, call this tint No. 2, and apply between the lines 2 and — 2 after No. 1 is dry. Add *three* portions of water to No. 2, call this tint No. 3, and apply between the lines 3 and — 3 after No. 2 is dry, etc. This method gives a banded appearance to the drawing.

In order to give a *uniformly* decreasing depth of tint proceed thus: First place the No. 0 tint over the portion in the shade as before and allow this tint to dry. Then between the lines 1 and — 1 apply tint No. 1, but instead of allowing it to dry “draw out” the edges with a brush moistened with water, and thus eliminate the sharp lines which would otherwise appear at 1 and — 1; after the “drawing out” is effected and the

tint is allowed to dry lay on tint No. 2, "draw out," etc. Either method is readily acquired after a few preliminary trials.

(*b*) Suppose an object placed between the sphere and the sun so as to cut off direct light from a small portion of the area included between the lines 5 and 7. This portion is now in shadow and must be shaded according to the statement of § 118*b*.

First, over the area inclosed by the line of shadow (or simply shadow) lay on tint No. 0. *Second*, lay on tints Nos. 1, 2, 3, 4, 5, 6, 7, as if the lines 5, 6, 7 were in the shade; *i. e.*, over 7 to 6 lay on tints Nos. 1, 2, 3, 4, 5, 6, 7, and over 6 to 5 lay on tints Nos. 1, 2, 3, 4, 5, 6. *Third*, over the shadow lay on a tint stronger than No. 0, call this tint No. S. *Fourth*, to tint No. S add one portion of water, call this tint No. 6', and lay on between 7 and 6; to tint No. 6' add two portions of water, call this tint No. 5', and lay on between 7 and 5, etc.

120. Shading of Any Surface. The proper shading for any surface may be determined from the sphere type (Plate XII, Fig. 49) by inscribing a tangent sphere in the surface and finding the intersections of the secant planes passed through the sphere with the surface.

Plate XII, Fig. 50, gives the shading lines on a vertical circular cylinder. A tangent sphere is inscribed in the cylinder, with center at the center of the base (on *H*) of the cylinder. Fifteen equidistant planes are passed perpendicular to a ray and their intersections with the base determined, thus giving the line 5, 6, 6, 5, 4, 3, 2, 1, 0, — 1, — 2, — 3, and — 4. It is at once evident that the lightest part of the cylinder is between 6 and 6 corresponding to the zone 6 on the sphere (§ 119).

Plate XII, Fig. 50*a*, gives the proper shading lines on a circular cylinder, axis parallel to *G. L.* *C . . . D*, with corre-

sponding shading lines, is for H projection and $E . . . F$, with corresponding shading lines, is for V projection.

121. Plate XII, Fig. 51, shows the shading on a sphere in V projection by the "drawing out" method; and Fig. 52 shows the shading on a vertical cylinder by the "banded" method.

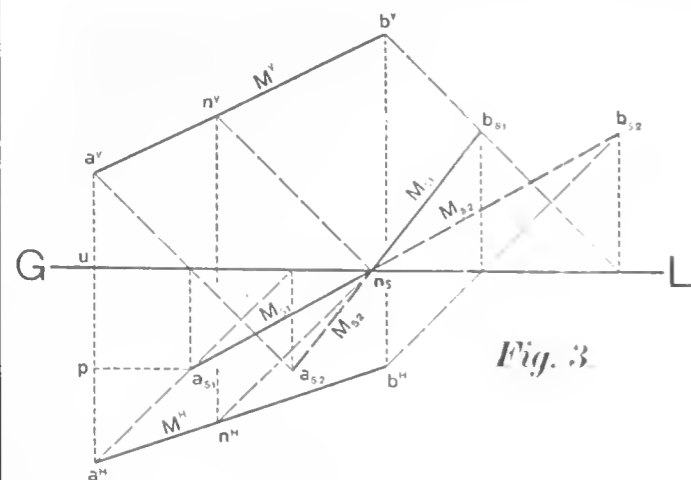


Fig. 3.

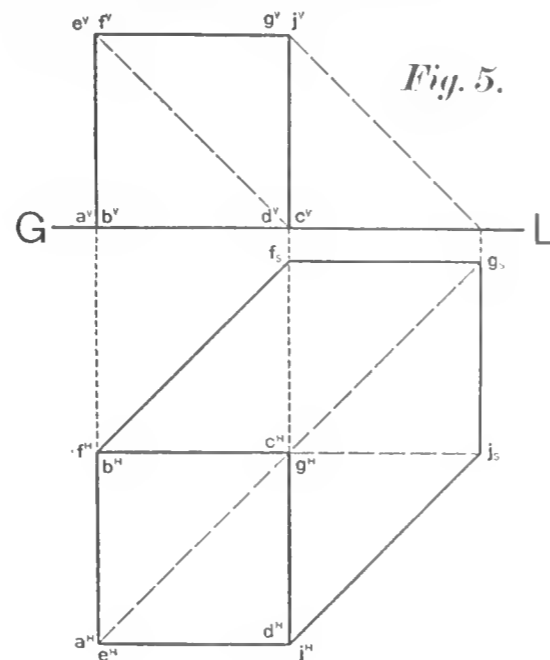


Fig. 5.

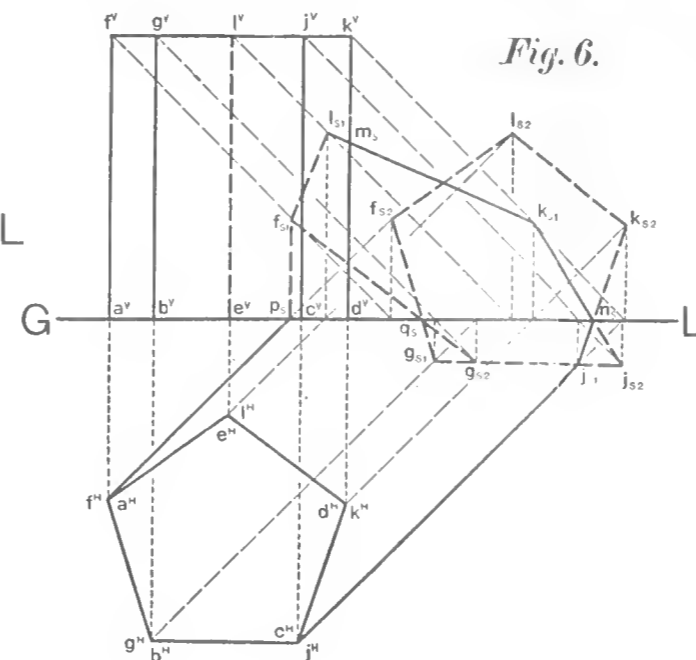


Fig. 6.

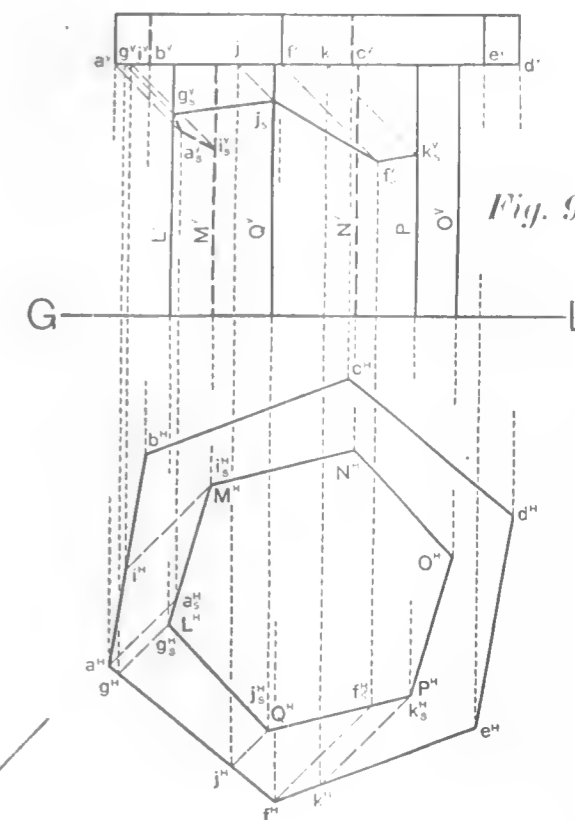


Fig. 9.

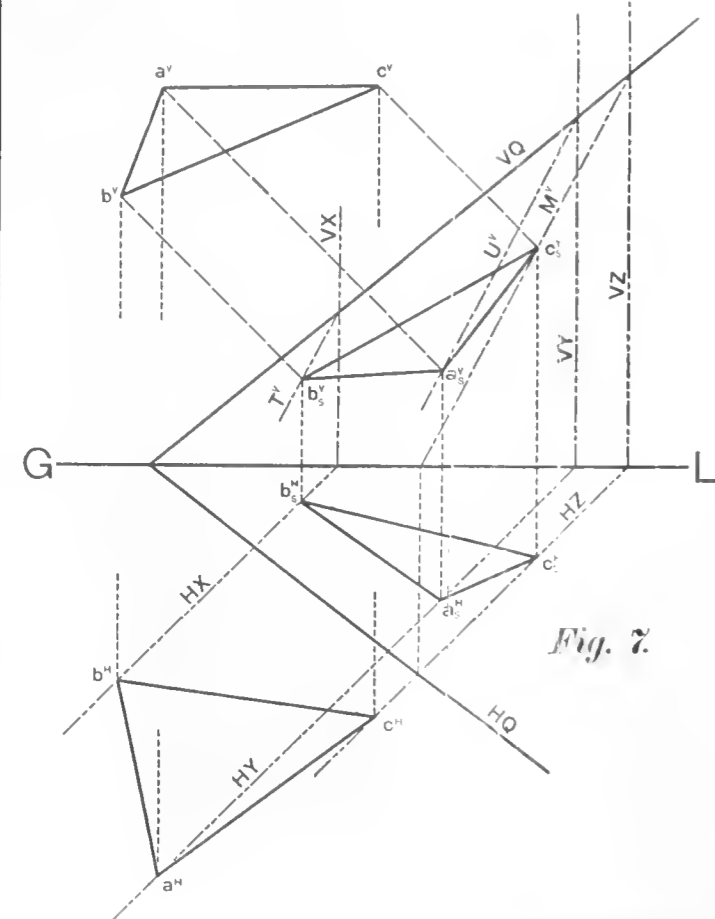


Fig. 7.

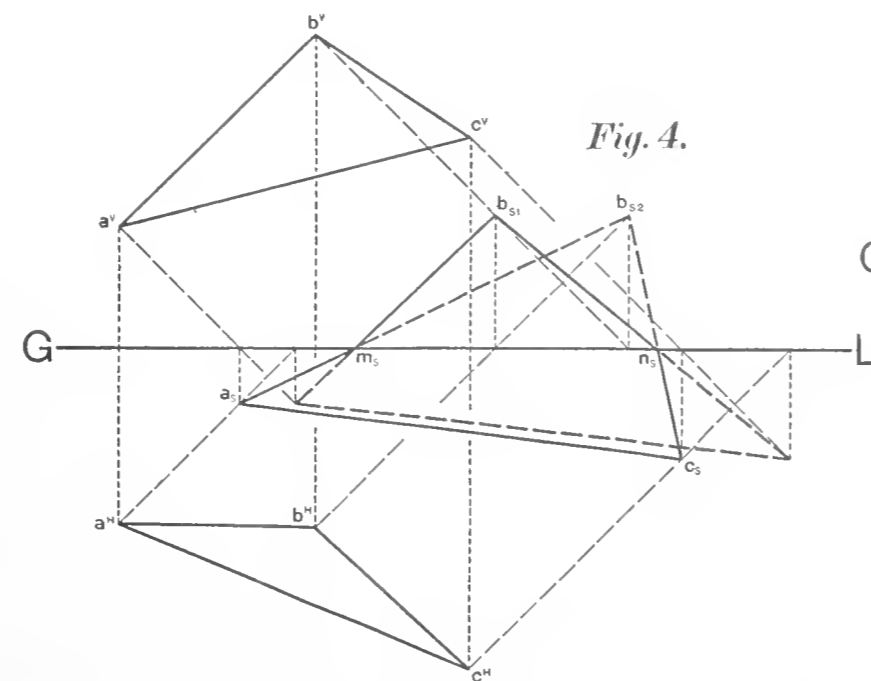


Fig. 4.

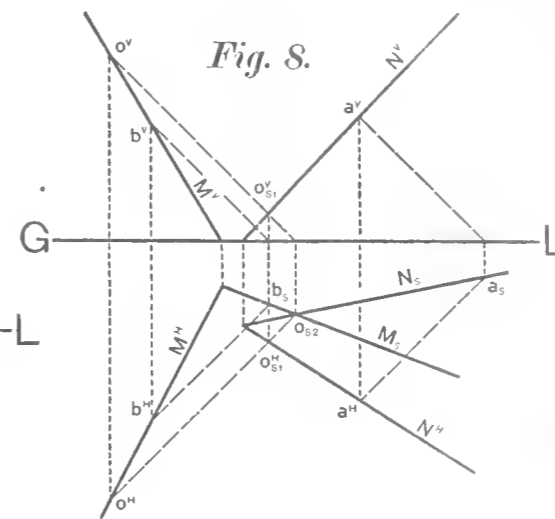


Fig. 8.

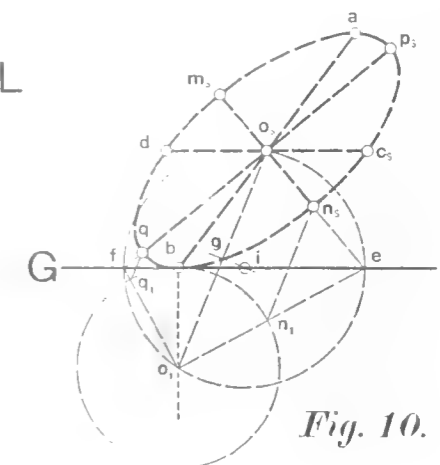


Fig. 10.

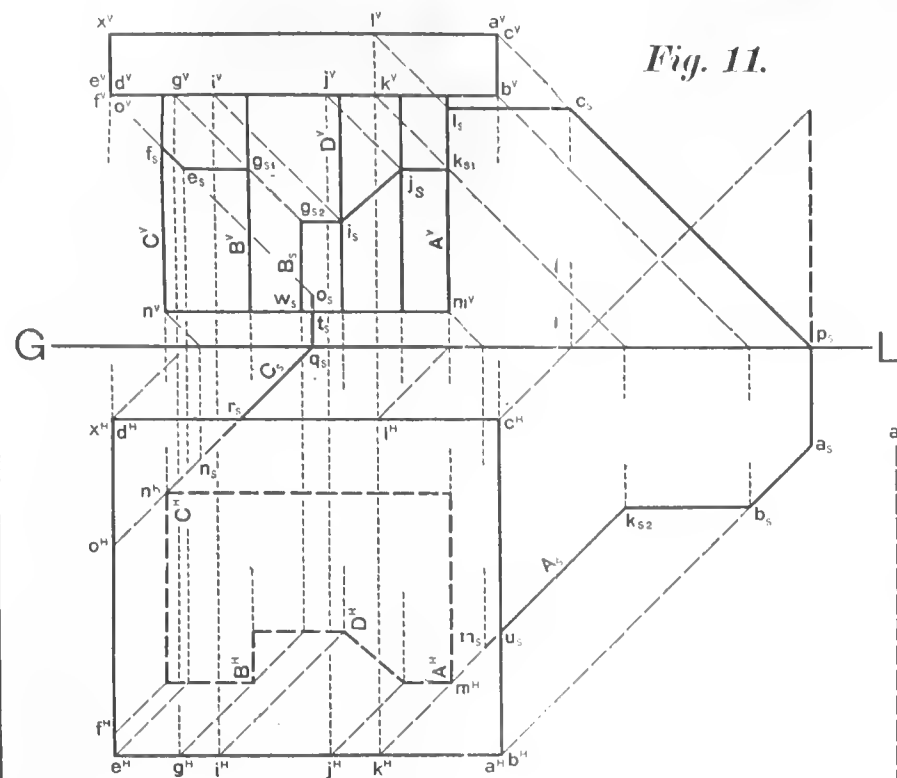


Fig. 11.

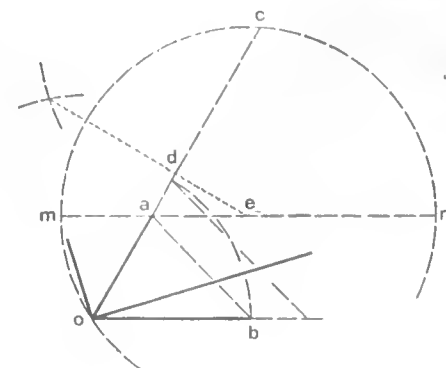


Fig. 15.

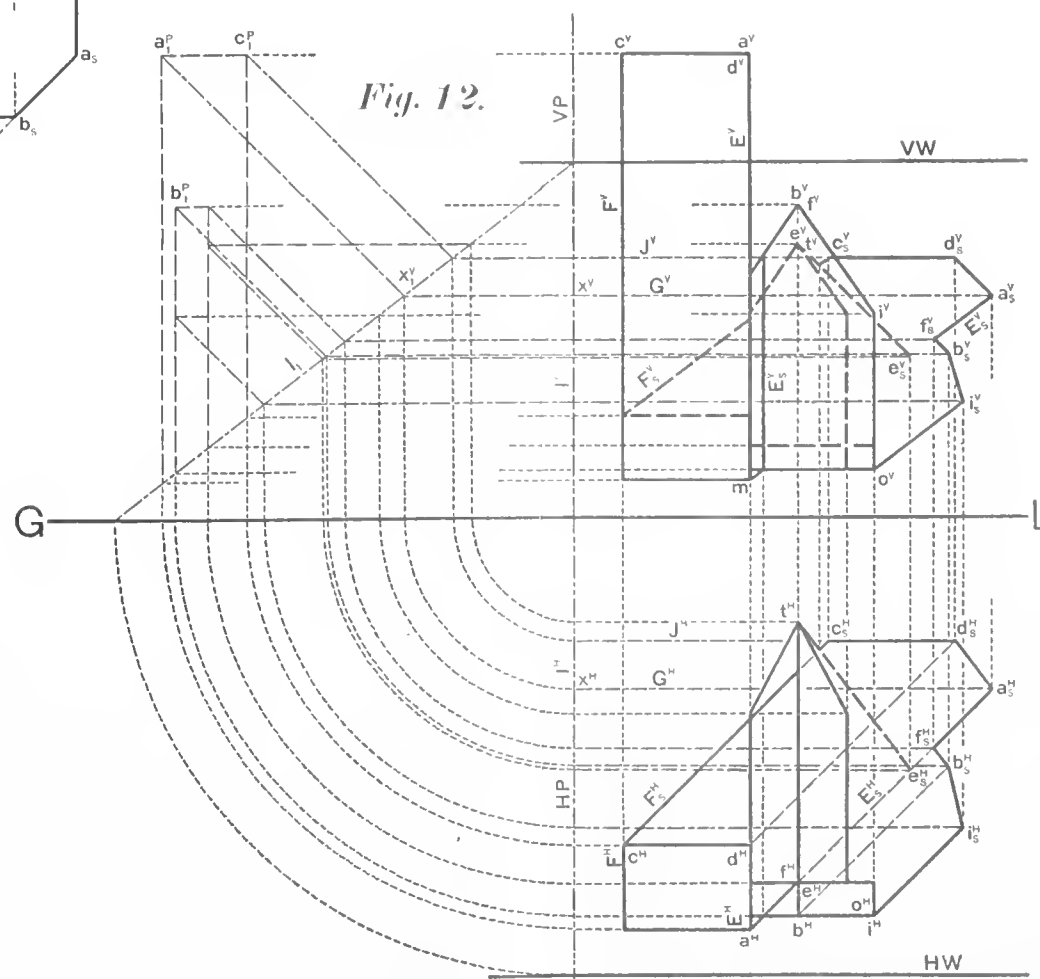


Fig. 12.

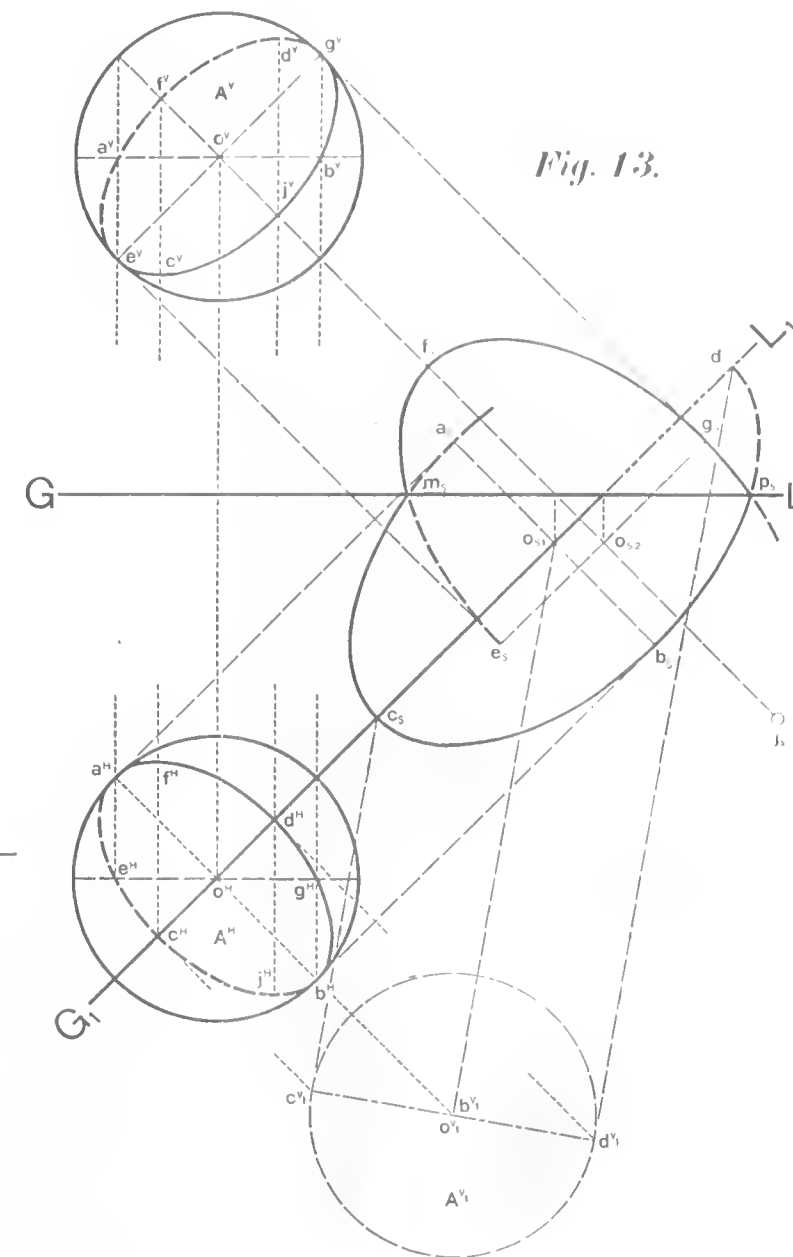


Fig. 13.

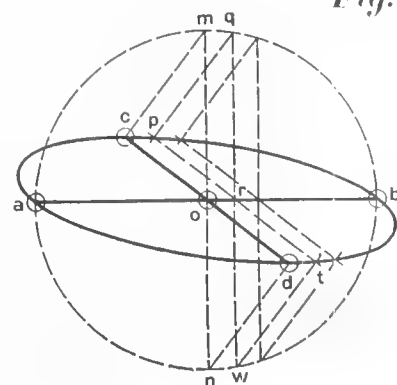


Fig. 14.

Fig. 17.

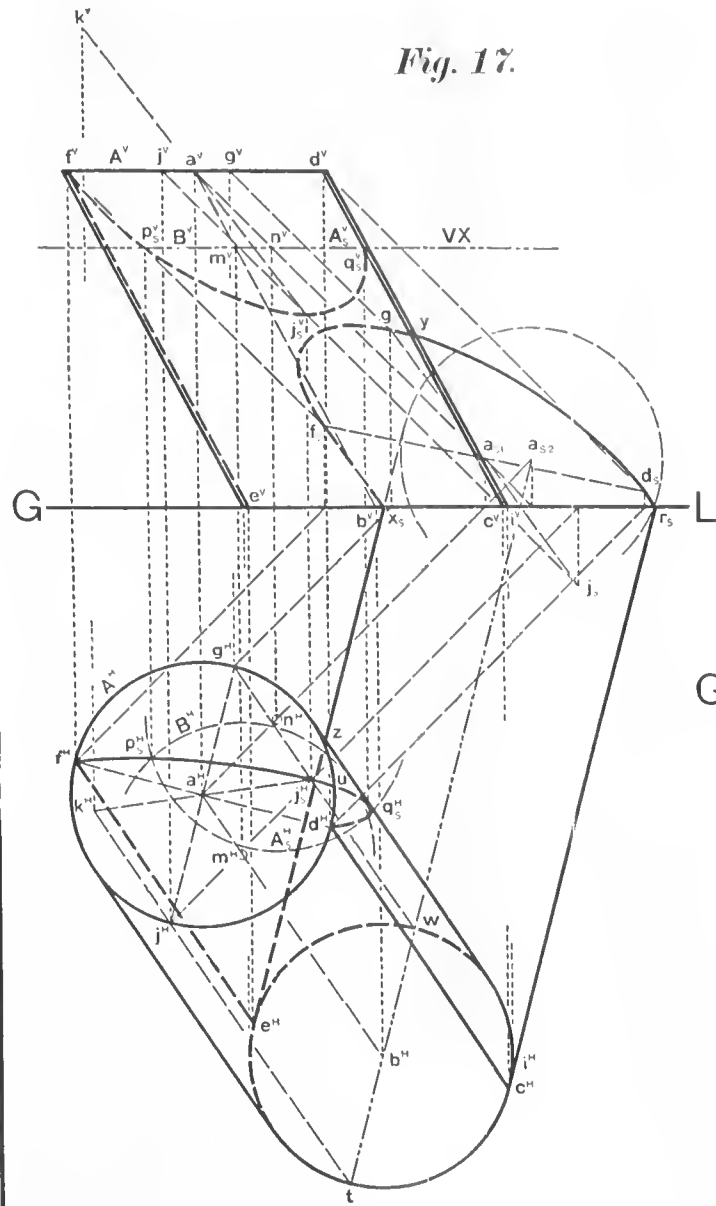


Fig. 18.

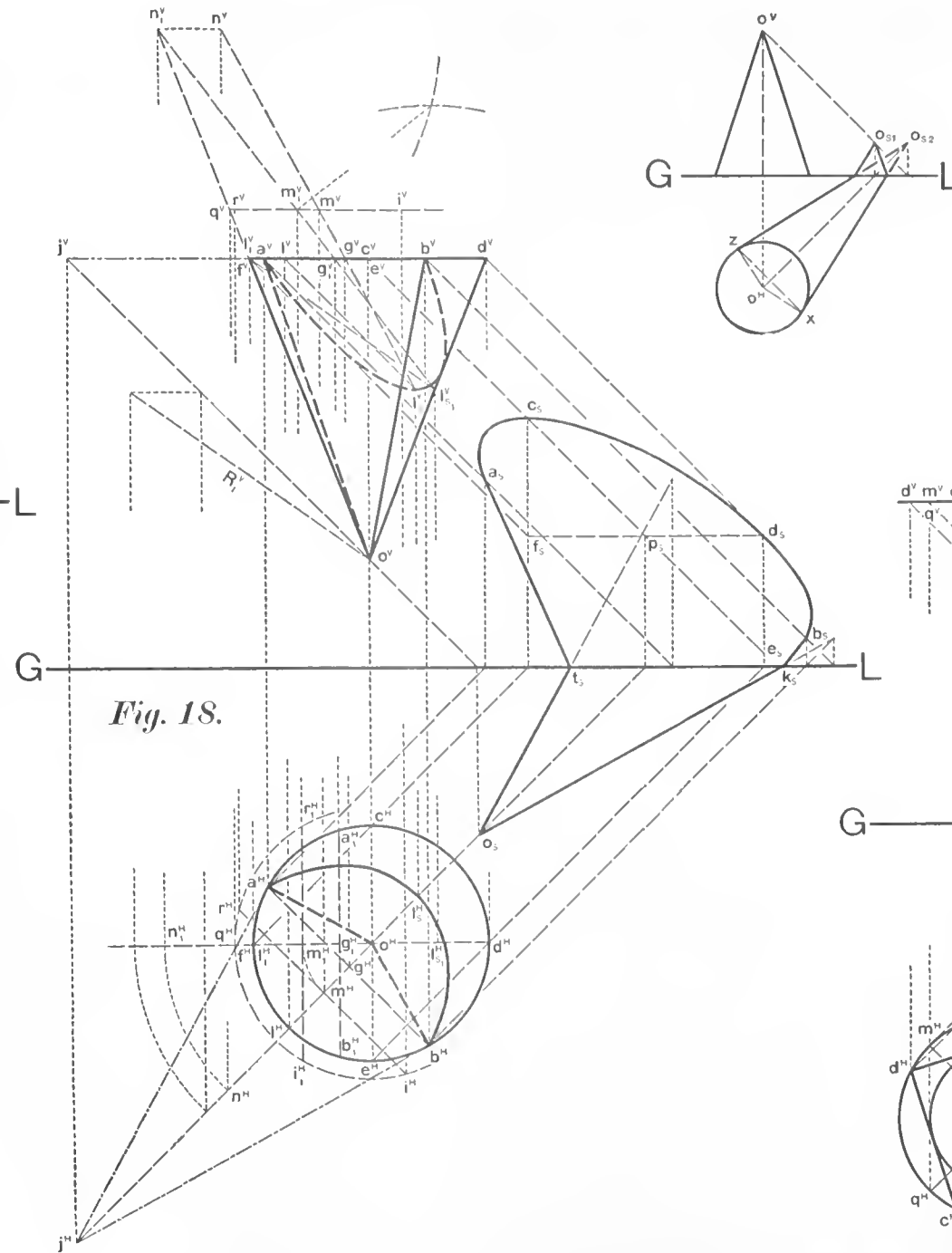


Fig. 16.

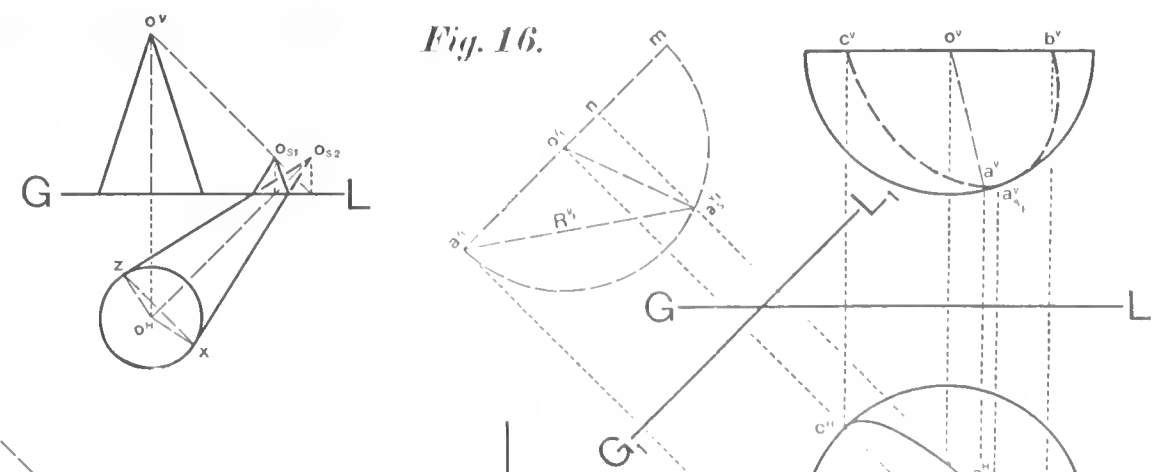
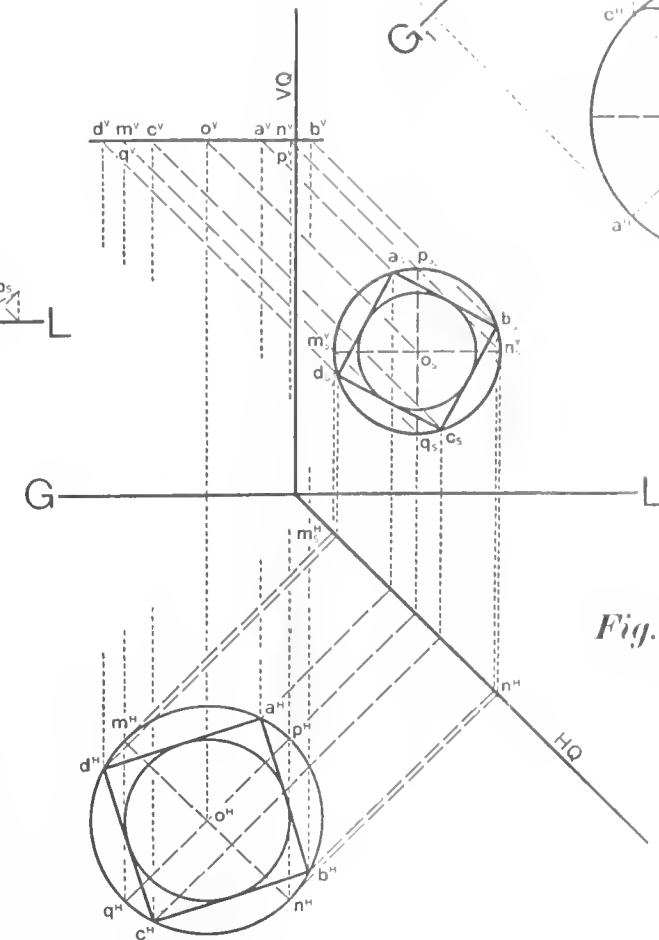
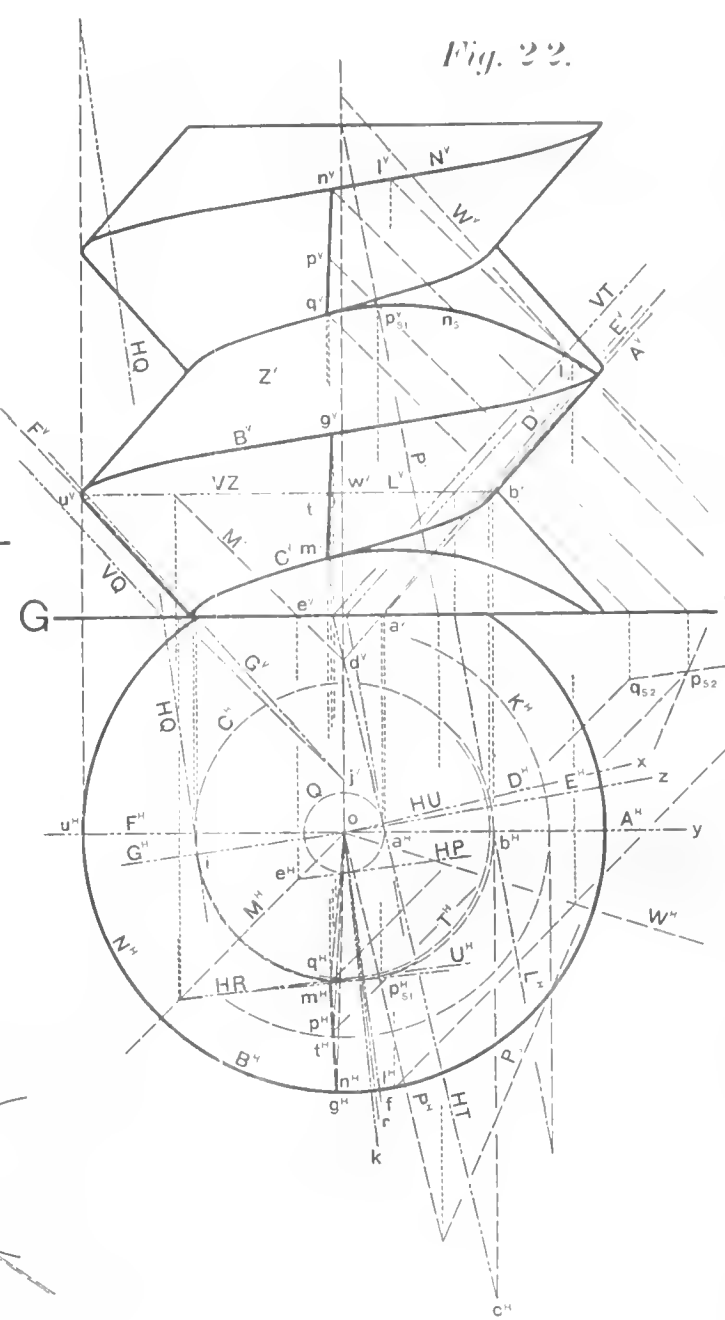
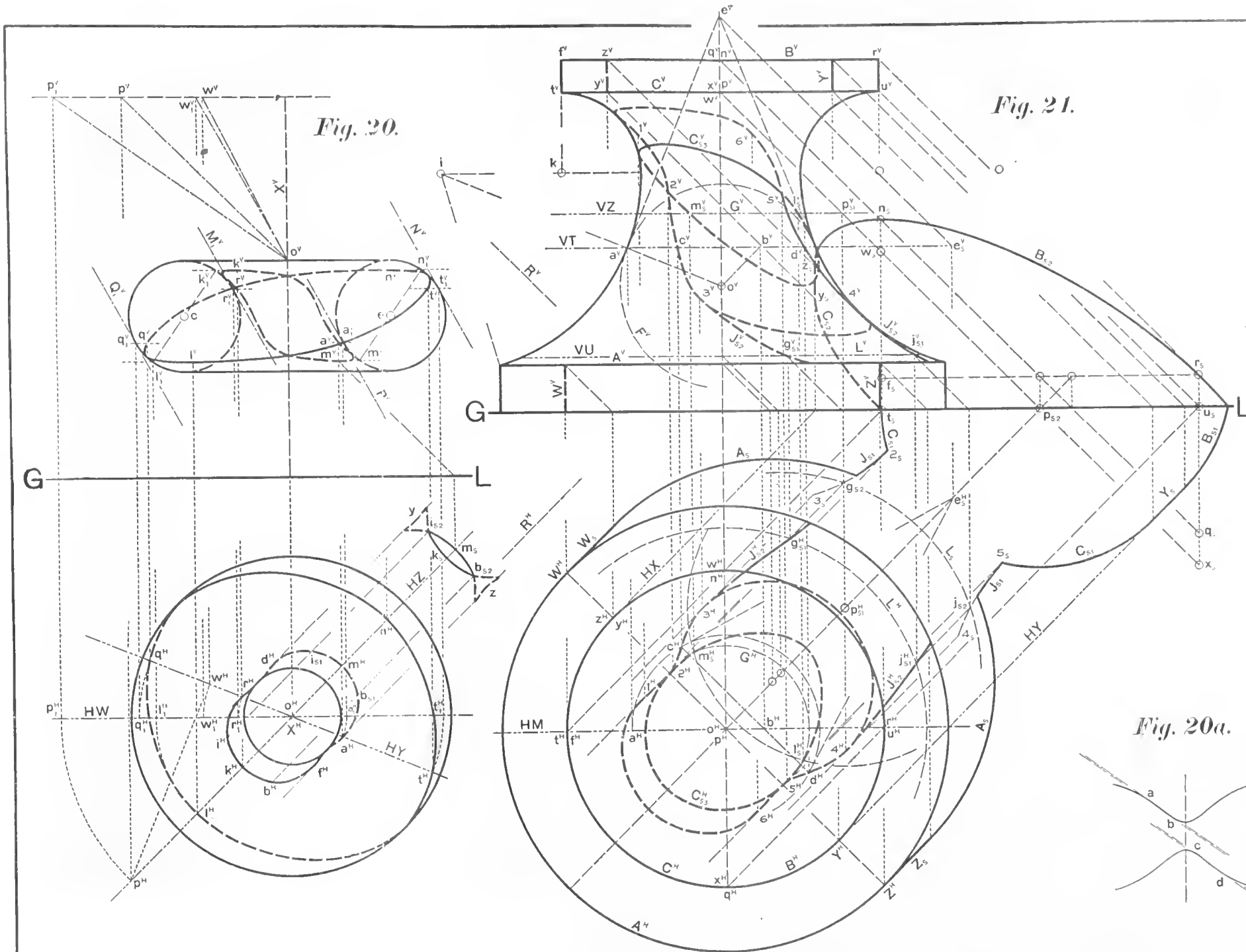


Fig. 19.





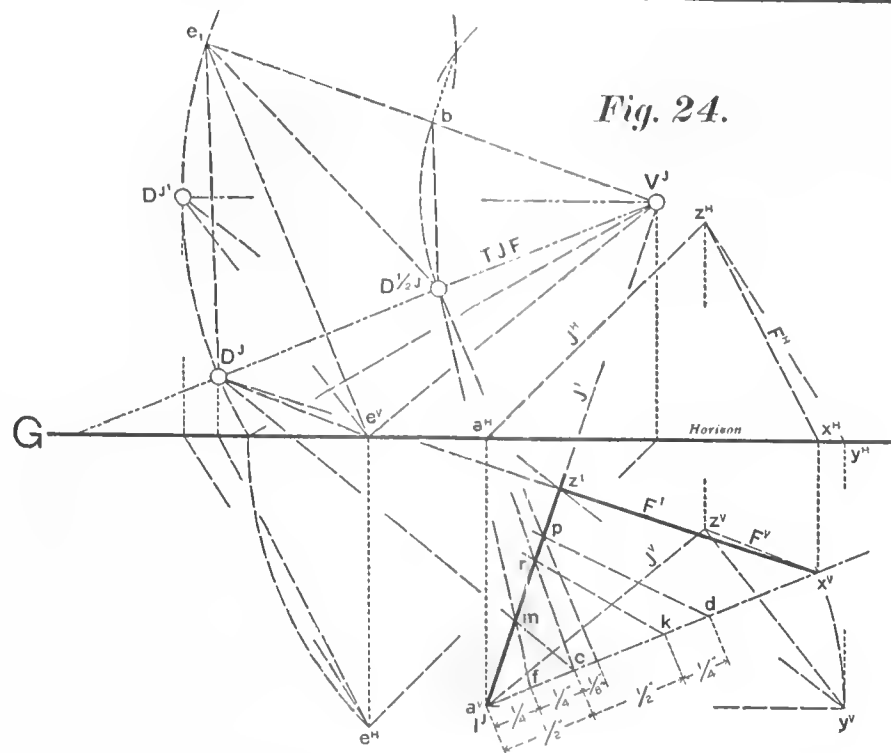


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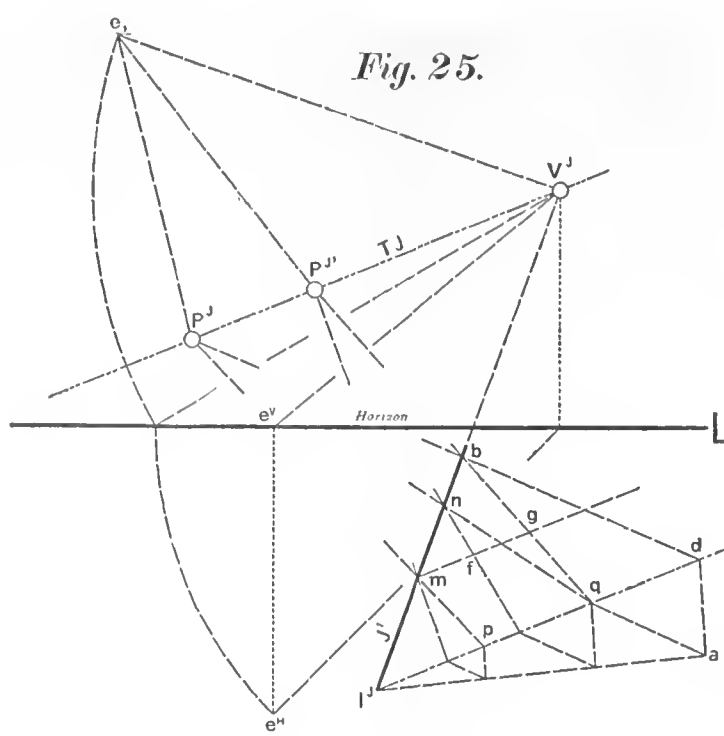


Fig. 25.

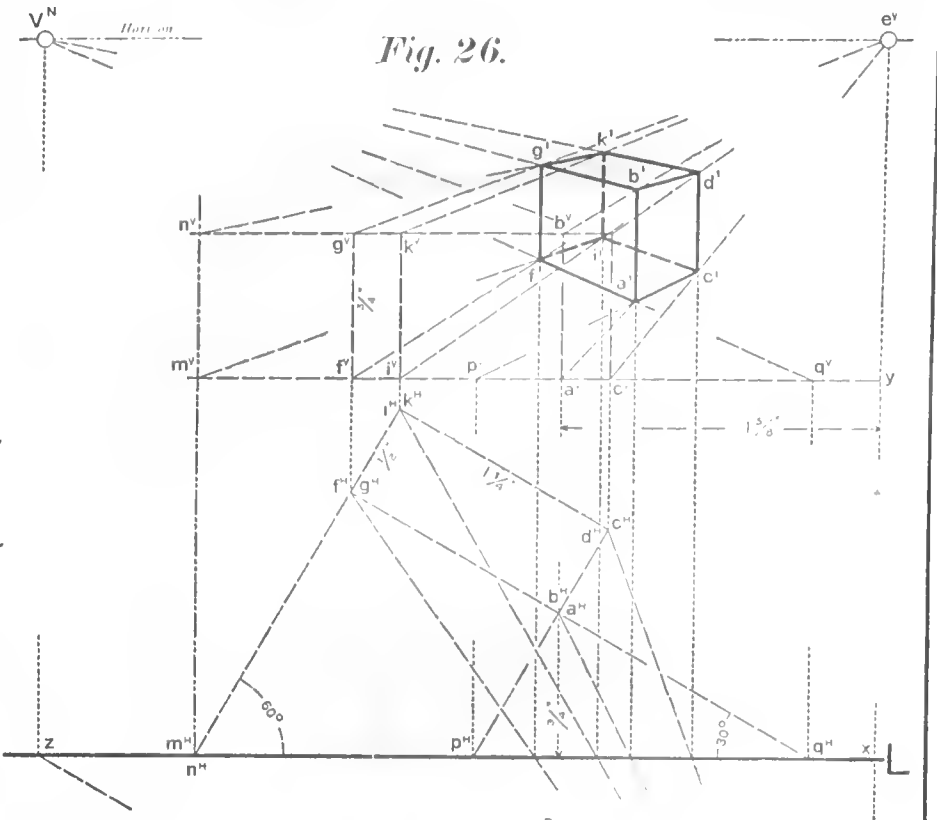


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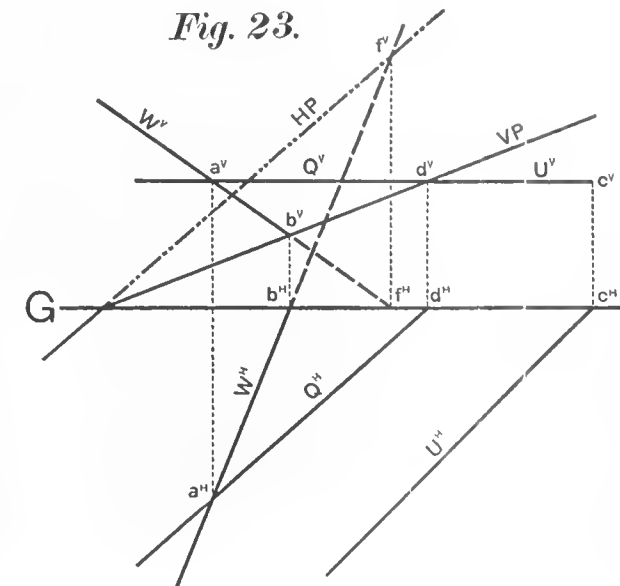


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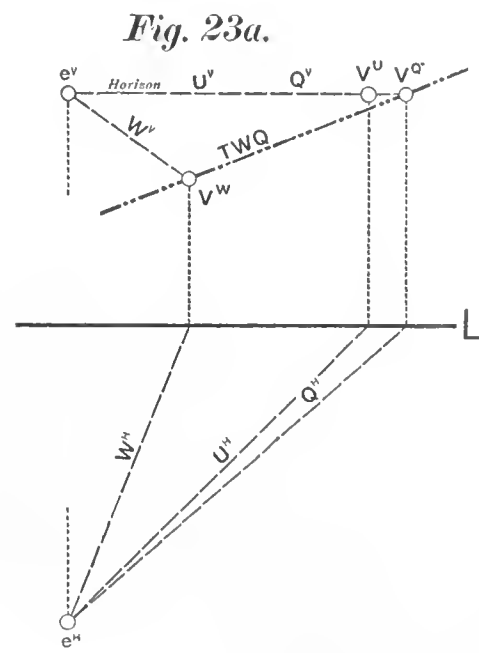


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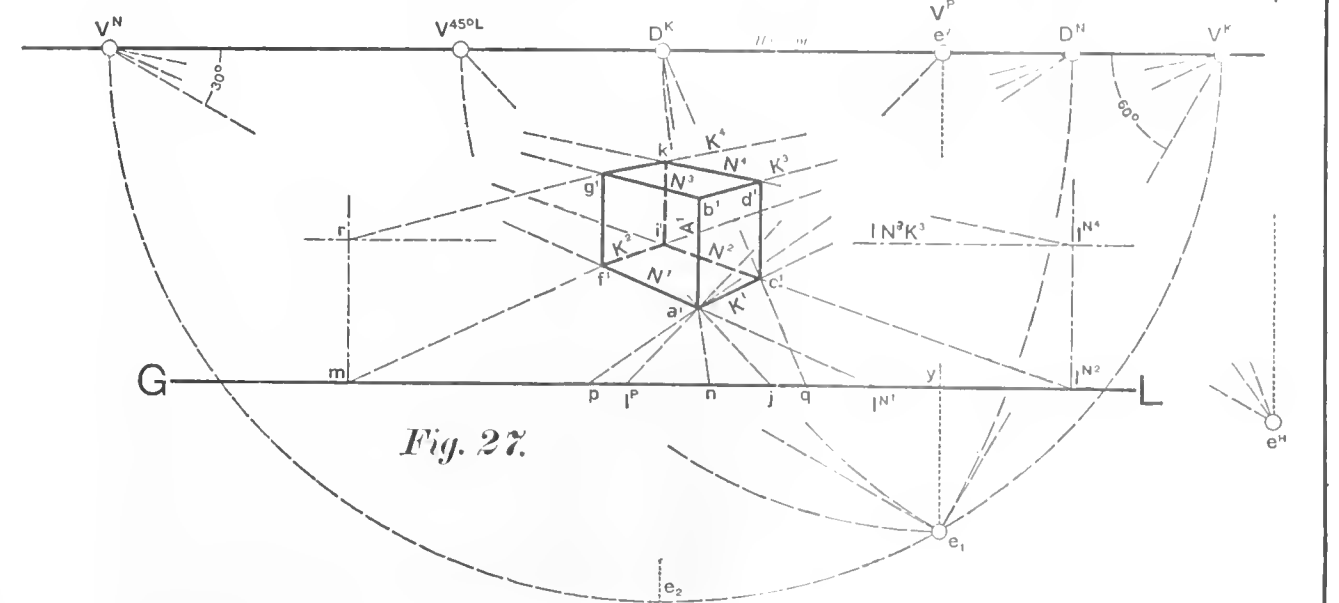


Fig. 27.

Fig. 28.

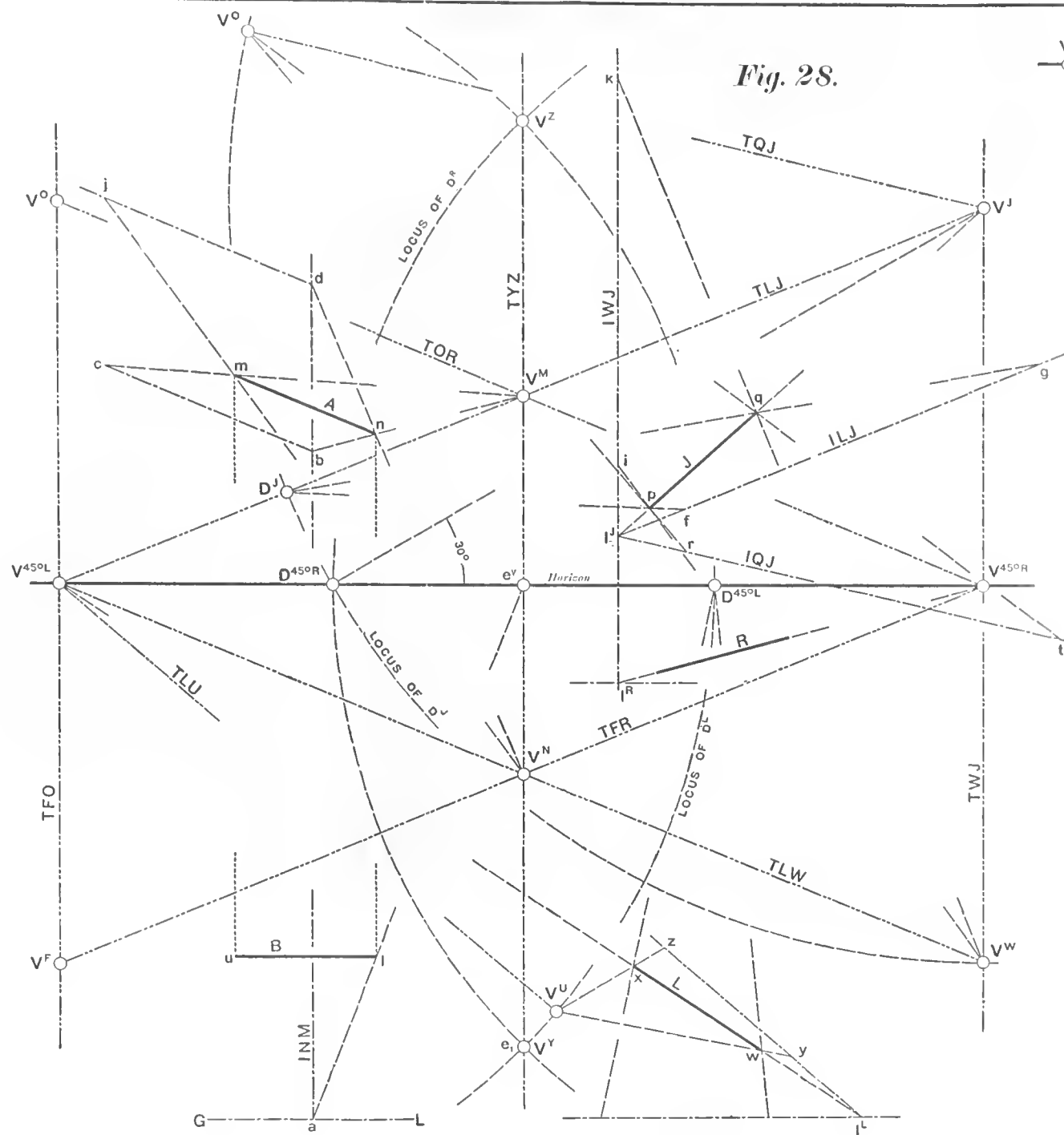
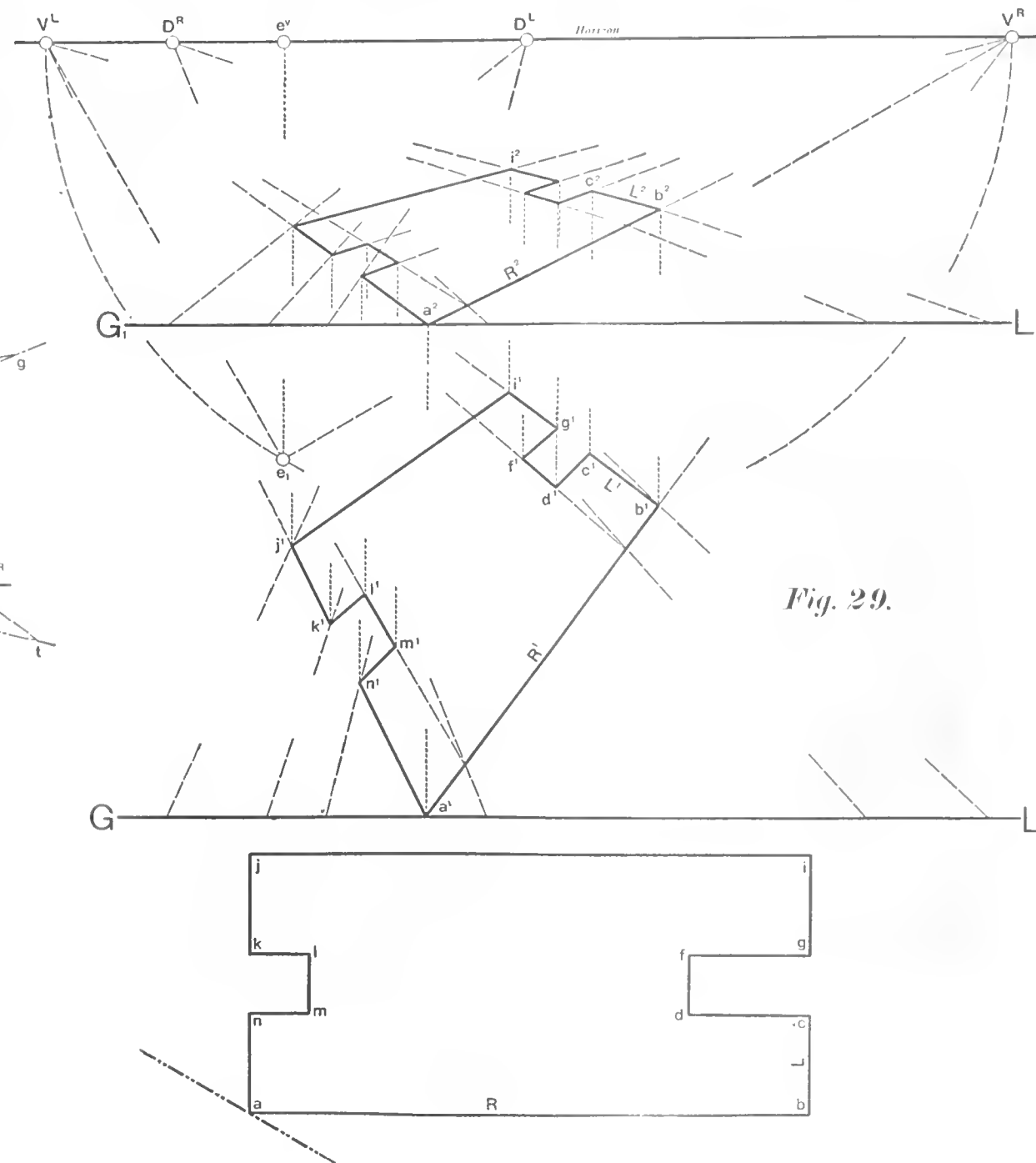


Fig. 29.



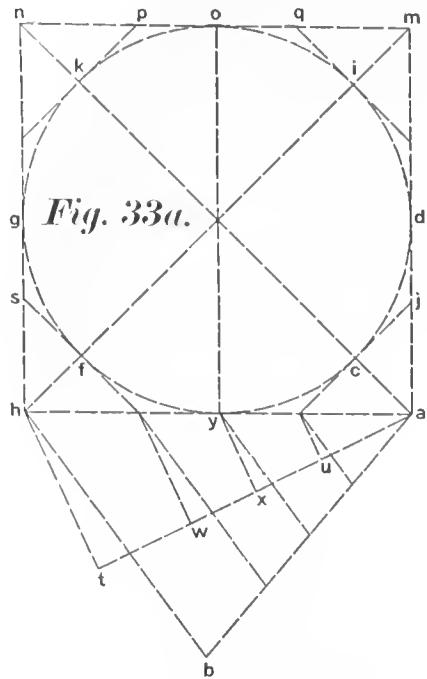


Fig. 33a.

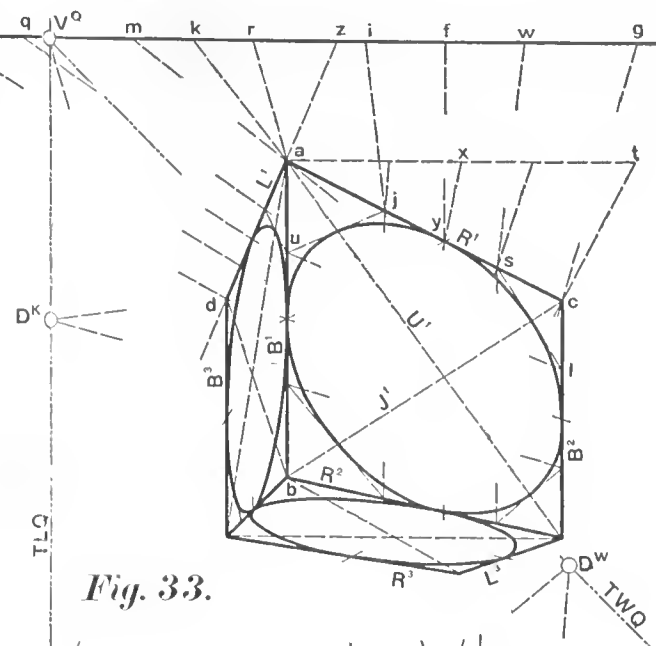


Fig. 33.

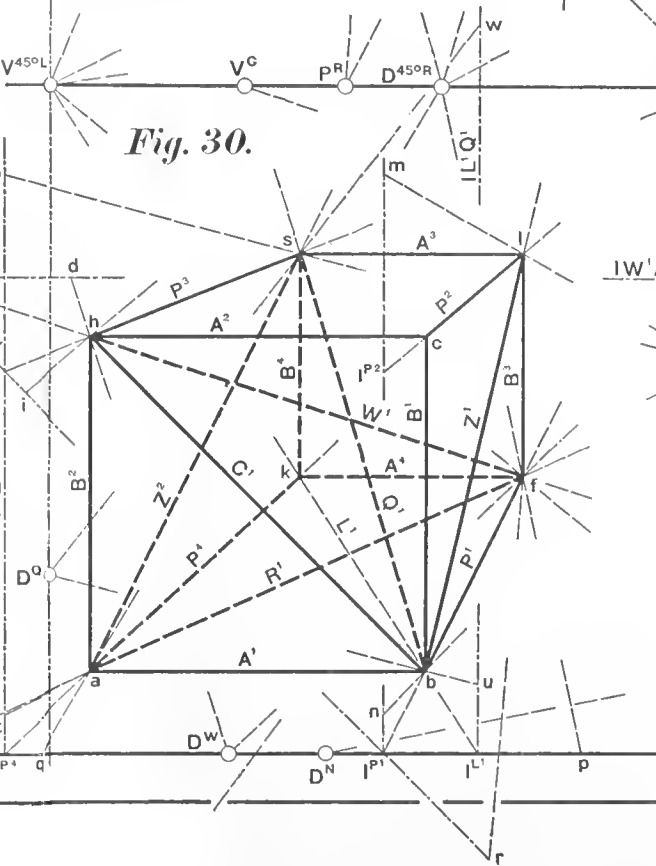


Fig. 30.

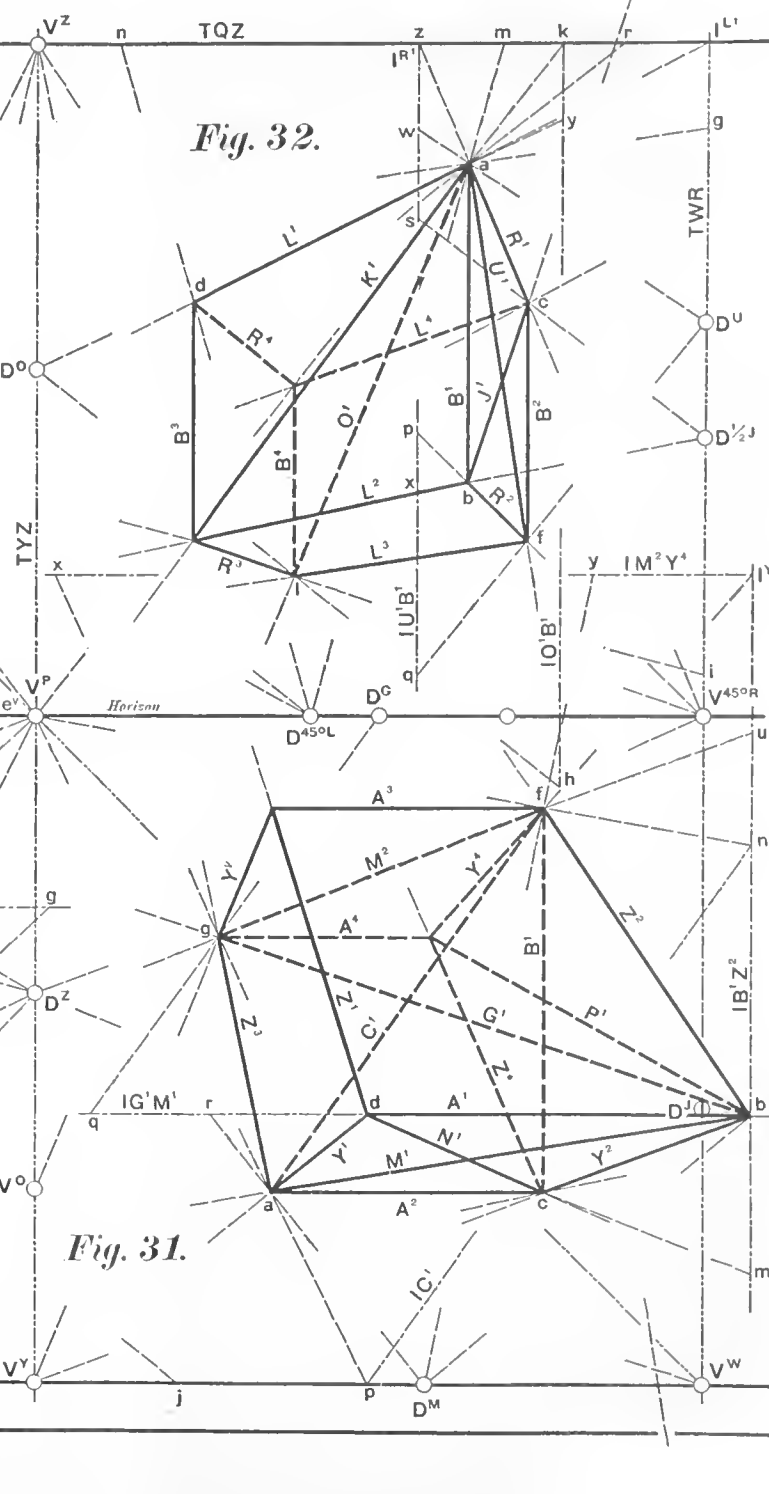


Fig. 31.

Fig. 32.

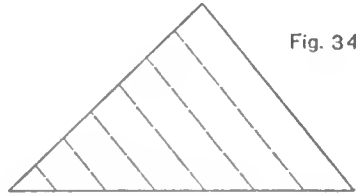


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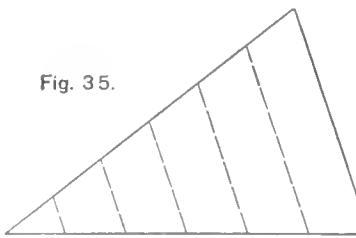


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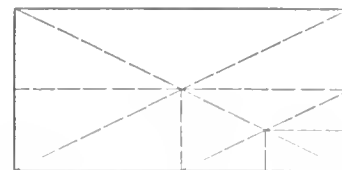


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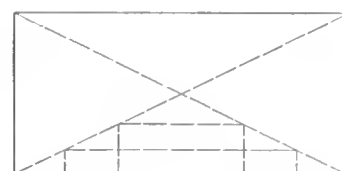


Fig. 37.

Fig. 43a.

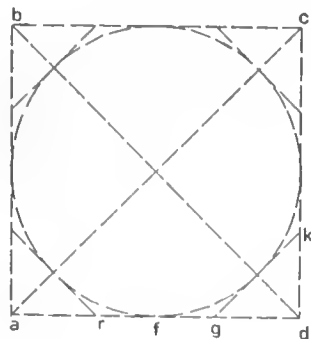


Fig. 40a.

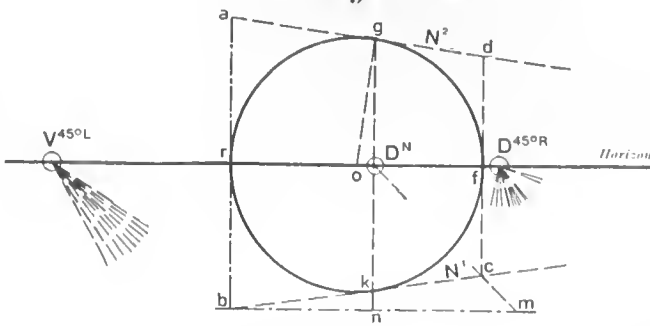


Fig. 10.

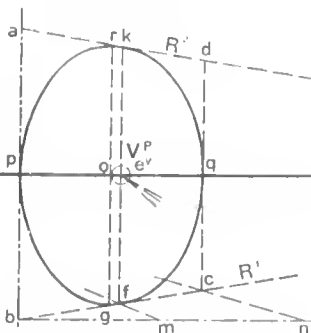


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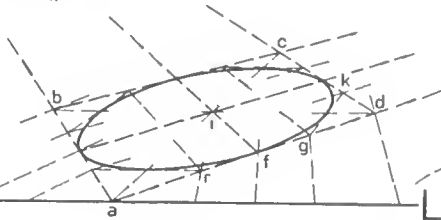


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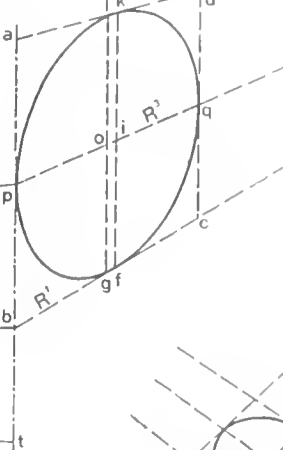


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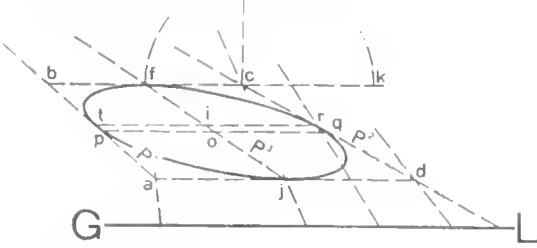


Fig. 39.

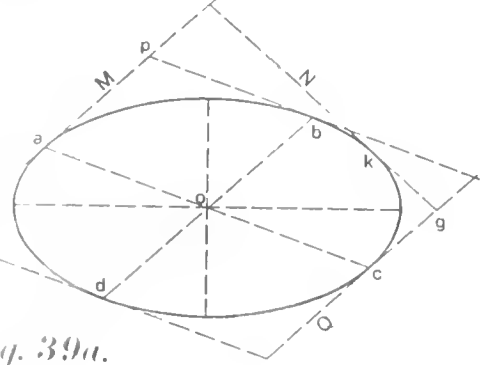


Fig. 39a.

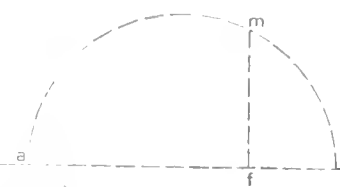


Fig. 38.

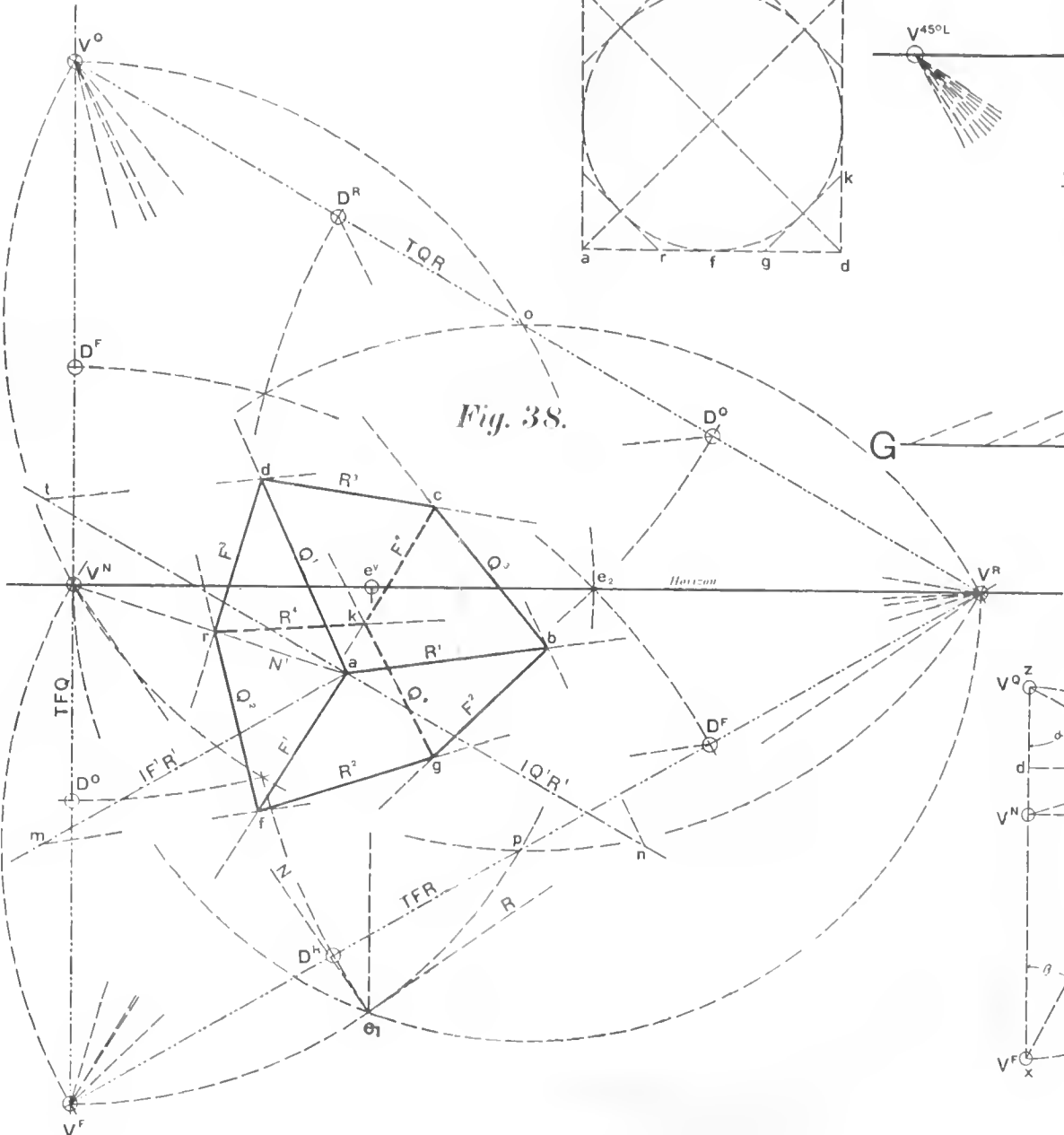


Fig. 38a.

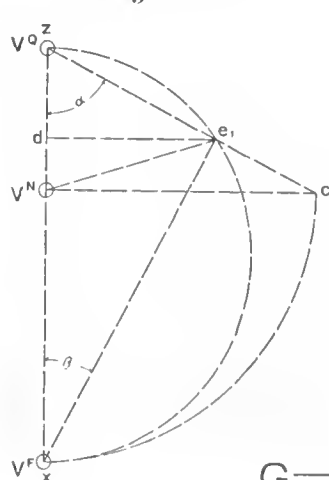
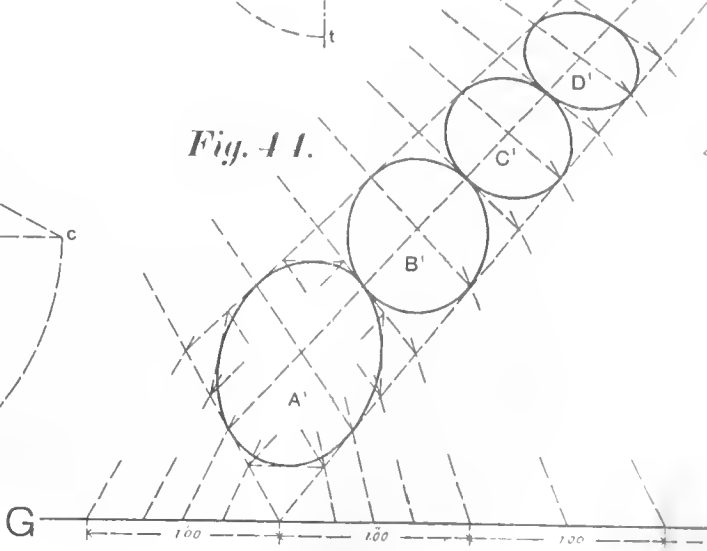


Fig. 41.



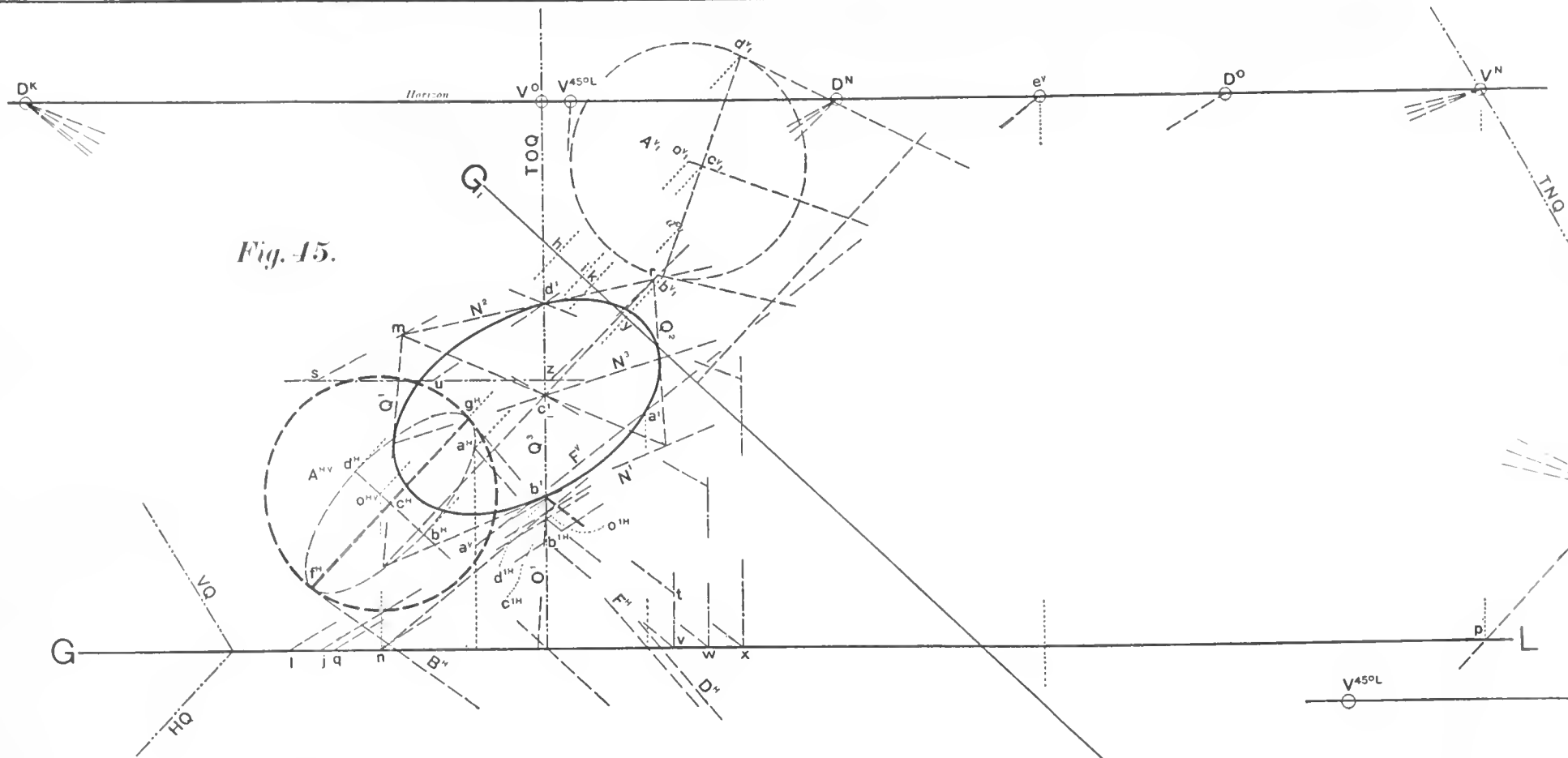


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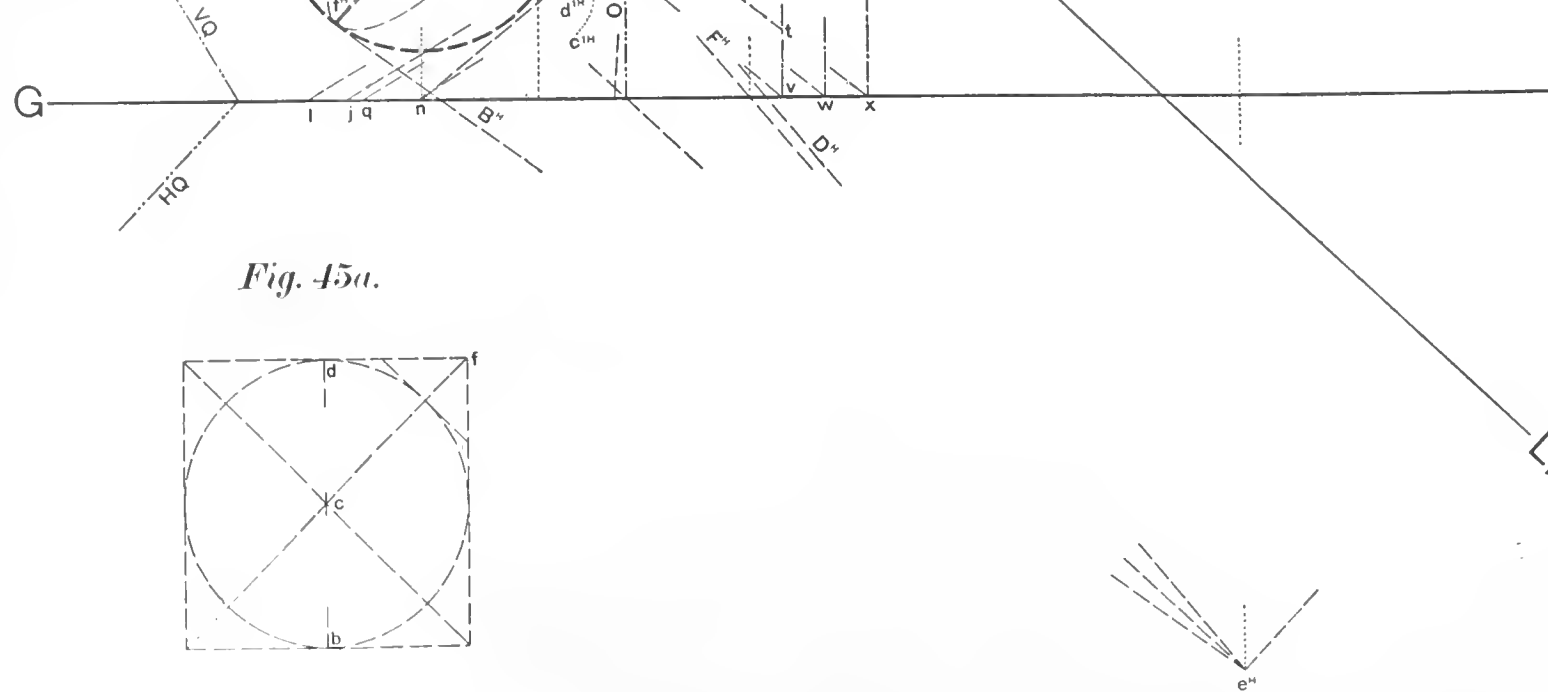


Fig. 45a.

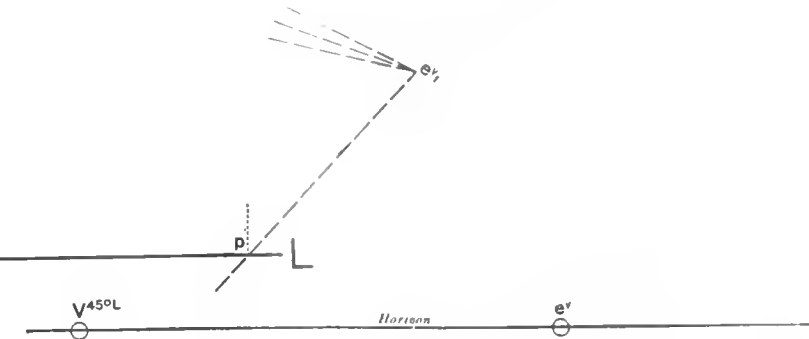
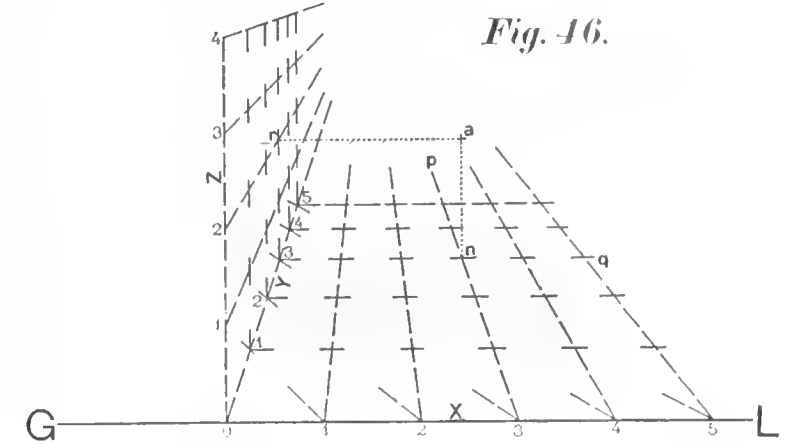
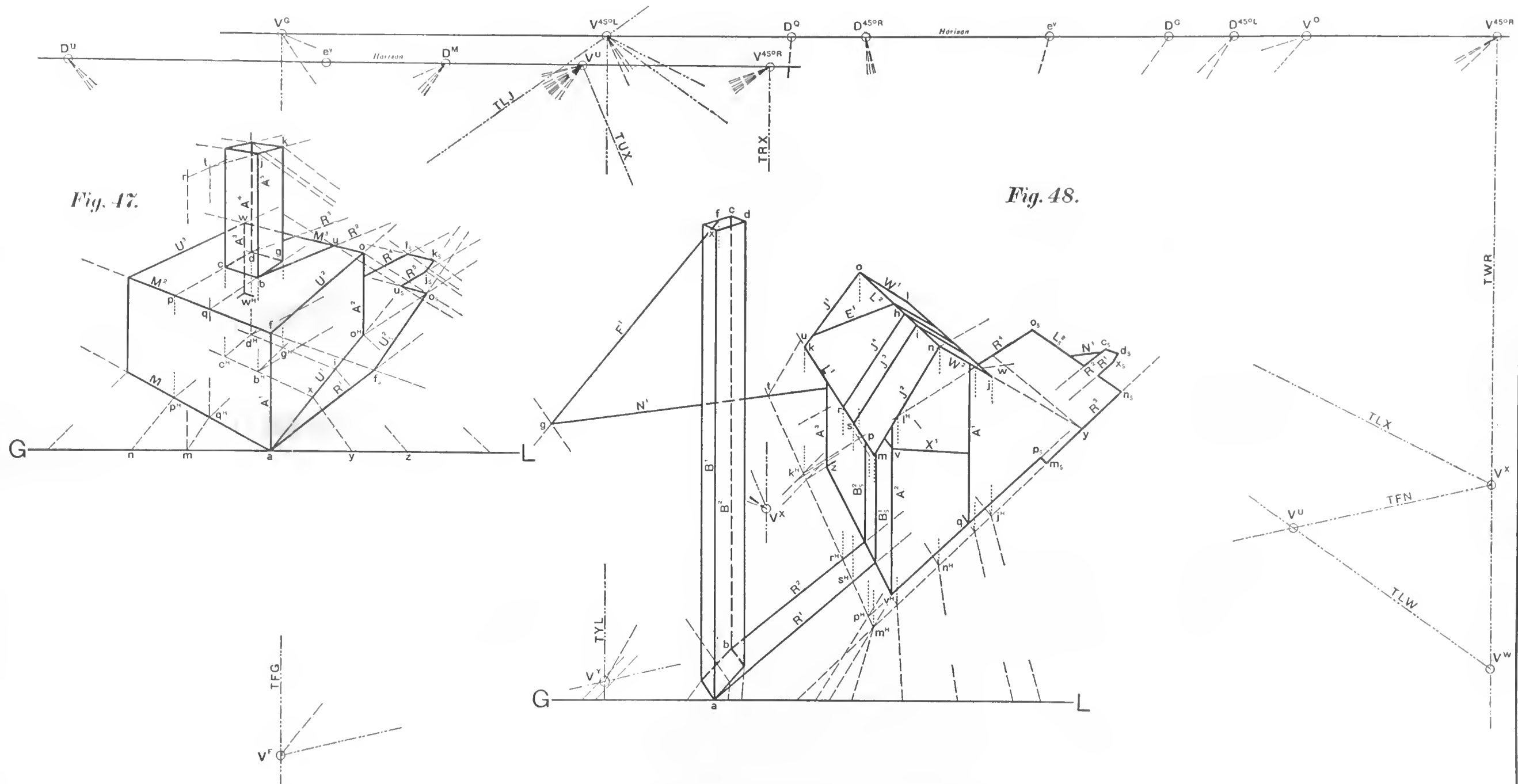


Fig. 46.





Station Point 2.5" above O and 13" in front of O

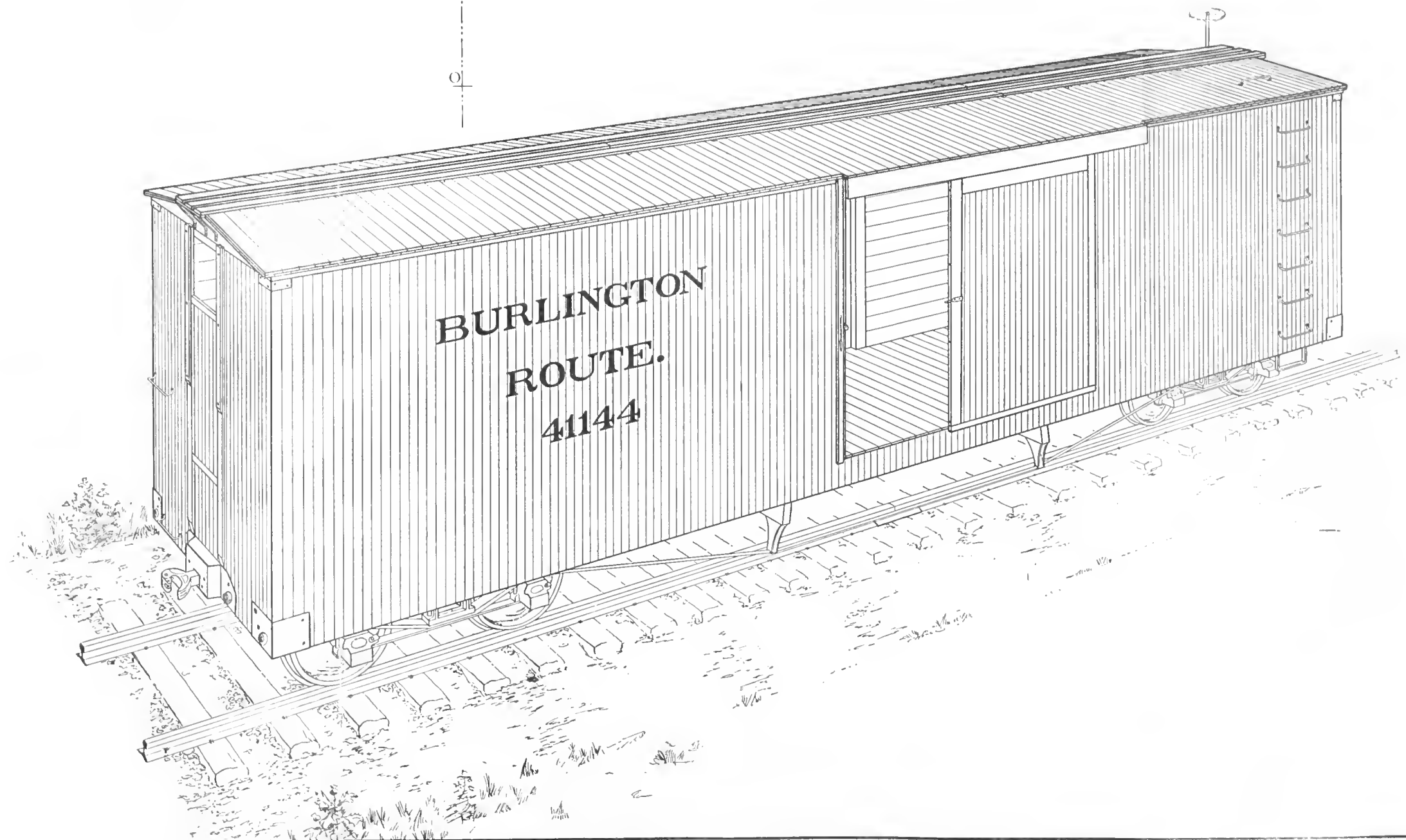


Fig. 49.

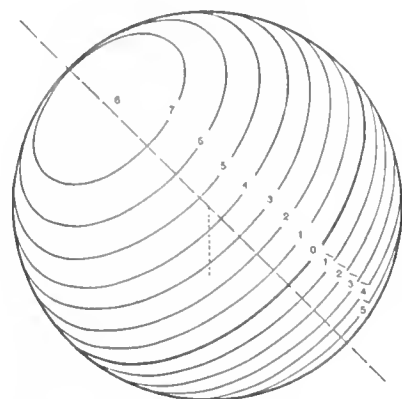


Fig. 49a.

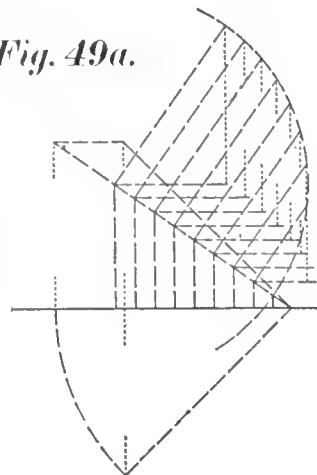


Fig. 51.



Fig. 52.



Fig. 50.

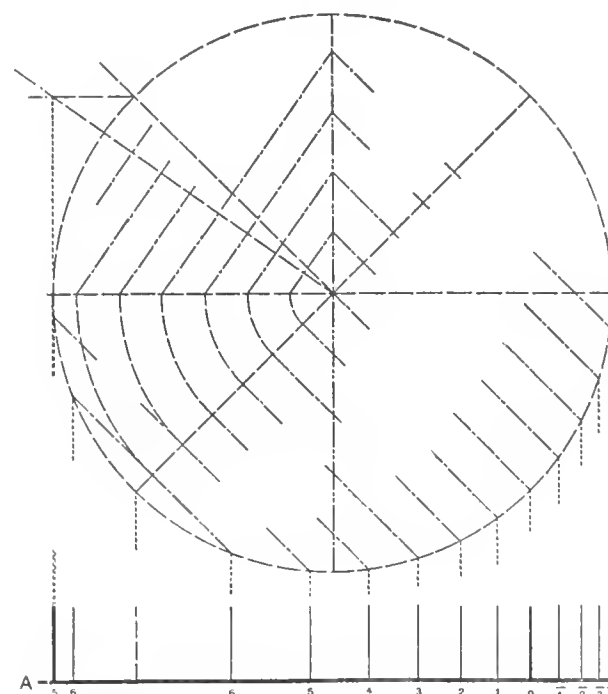
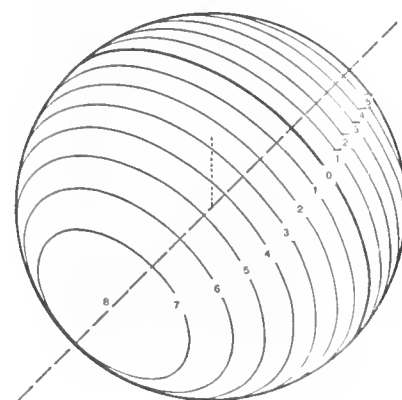
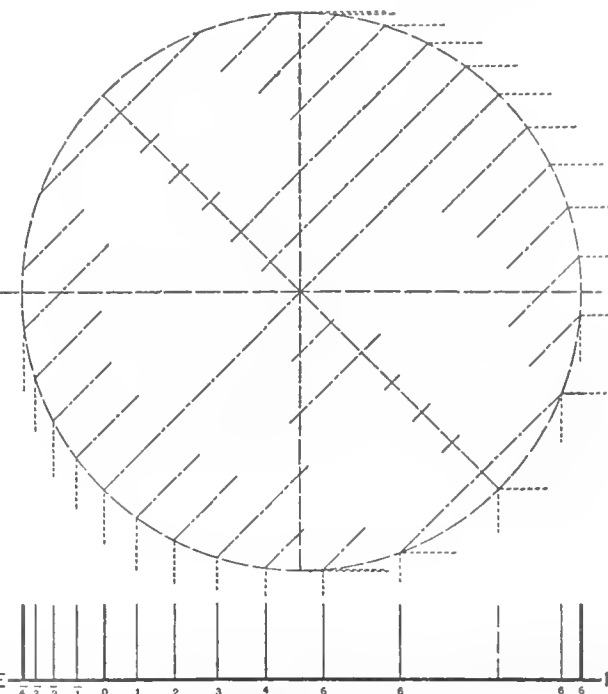


Fig. 50a.



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